ELSEVIER

Journal of Empirical Finance 4 (1997) 373–401

Journal of EMPIRICAL FINANCE

Another look at long memory in common stock returns

Craig Hiemstra [a,*], Jonathan D. Jones [b]

[a] *Department of Accounting and Finance, University of Strathclyde, Glasgow, G4 OLN, UK*
[b] *Office of Economic Analysis, Securities and Exchange Commission, Washington, DC 20549, USA*

Abstract

We apply the modified rescaled range test to the return series of 1,952 common stocks. The results indicate that long memory is not a widespread characteristic of these stocks. But logit models of the event of a test rejection reveal that rejections are linked to firms with large risk-adjusted average returns. The maximal moment of a return distribution is also found to influence the event of a rejection, but not in a way suggestive of moment-condition failure. Evidence suggestive of survivorship bias is also uncovered. We conclude that there is some evidence consistent with persistent long memory in the returns of a small proportion of stocks. © 1997 Elsevier Science B.V.

JEL classification: C22; G12

Keywords: Long memory; Rescaled range test; Moment condition failure; Survivorship bias; Common stock returns

1. Introduction

A stationary long-memory process can be characterized by its autocorrelation function which decays at a hyperbolic rate. Such a decay rate is much slower than the geometric rate of weakly-dependent processes such as finite-order stationary ARMA processes. The fractionally-integrated ARMA models of Granger (1980), Granger and Joyeux (1980), Hosking (1981), and Mandelbrot and Van Ness

* Corresponding author. E-mail: c.hiemstra@strath.ac.uk, jonesjd@aol.com.

PII S0927-5398(96)00016-3

(1968) display long memory in the means of their corresponding series. Recently, some models have been introduced to describe long memory in variances. These are the fractionally-integrated GARCH models of Baillie et al. (1993) and Bollerslev and Mikkelsen (1993), and the long-memory stochastic volatility models of Breidt et al. (1994) and Harvey (1993). These models allow for the presence of long memory in the squared innovations of a time series.

It is the fractional-integration parameter, $-0.5 < d < 0.5$, in long-memory models which determines the presence and describes the nature of the long memory. For $d \neq 0$, a process exhibits long memory. The nature of the long memory depends on the sign of d. For $d > 0$, the sum of the autocorrelations diverges to infinity, the dependence is positive, and the long memory is called persistent. Whereas for $d < 0$, the sum of the autocorrelations converges to zero, the dependence is negative, and the long memory is called antipersistent. [1] More precise details of certain fractionally-integrated processes are presented in Section 2. [2]

The issue of whether or not the means of financial-asset returns possess long-memory components is an empirical question with important implications for equilibrium asset-pricing models. These models are characterized by the absence of arbitrage opportunities. Mandelbrot (1971) shows under certain conditions that perfect arbitrage, the process yielding the absence of arbitrage opportunities, is impossible when the process of arbitrage is applied to a price series possessing increments which are driven by innovations with long memory. For such cases, the process of arbitrage does not generate perfect arbitrage; instead, a market generating asset returns with long memory components in their means can be grossly inefficient. Hodges (1995), for example, shows in certain cases that essentially riskless arbitrage can be accomplished in a market where prices possess long-memory components.

Common methods of testing for the presence of long memory in a series are the modified rescaled range (or, R/S) test of Lo (1991) and the test of Geweke and Porter-Hudak (1983). Hereafter we will refer to these tests as MRS and GPH. [3] Results based on these tests of long memory in asset-return series are mixed but

[1] An alternative characterization of a long-memory process focuses on its spectral density at the origin. The spectral density at the origin respectively diverges to infinity and converges to zero for persistent and antipersistent long-memory processes.

[2] See also Baillie (1995) and Brock and de Lima (1995, Sctn.3). These papers contain recent surveys of long-memory models and evidence of such behavior in economic and financial time series.

[3] Some other tests for long memory can be found in Davies and Harte (1987) and Robinson (1991). A Monte Carlo study fo the finite-sample size and power of the MRS, GPH, and other long-memory tests is contained in Cheung (1993). See also Agiakloglou and Newbold (1994).

largely negative. [4] Studies where some combination of these tests are applied to asset-return series include Ambrose et al. (1993), Cheung and Lai (1993), Cheung et al. (1993), Crato (1994), Goetzmann (1993), Lo (1991), and Mills (1993). Some of the test statistics in these studies indicate evidence of long memory in weekly UK gold returns (Cheung and Lai), weekly aggregate returns on Italian, Japanese, and West German stocks (Crato), daily UK aggregate returns (Mills), and annual UK aggregate returns (Goetzmann). Alternatively, no strong statistical evidence of long memory is found in certain daily, weekly, and monthly US aggregate stock returns (Abrose, Ancel, and Griffiths; Crato; Goetzmann; and Lo). Similarly, Crato finds no evidence of long memory in weekly aggregate returns on Canadian, French, and UK stocks. And lastly, Cheung, Lai, and Lai find no evidence of long memory in aggregate monthly returns on German, Italian, Japanese, and UK stocks. With the exception of Crato (for a West German stock index) and Goetzmann, the authors of these studies draw overall conclusions against the presence of long memory in the series which they investigate.

The conclusions drawn from the majority of the studies mentioned above are in contrast with those drawn from many studies of long memory in asset returns based on R/S analysis, a method first introduced by Hurst (1951) and further developed in the studies of Mandelbrot and Wallis (1969) and Wallis and Matalas (1970) among others. This type of analysis is regression based, where a sample of the logarithms of rescaled range statistics over varying-length subperiods of a series is regressed on the logarithm of the subperiod lengths. A regression coefficient on the logarithm of the subperiod lengths differing from 0.5 can be suggestive of long memory in the series. Values between 0 and 0.5 can be indicative of antipersistent long memory, while values between 0.5 and 1 can indicate persistent long memory.

Studies in which R/S analysis is applied to financial time series include Greene and Fielitz (1977), Booth and Kaen (1979), Booth et al. (1982a,b), Helms et al. (1984), and more recently Peters (1989, 1991, 1992, 1994). The authors of these studies respectively conclude that long memory is present in return series relating to common stocks; gold; gold and foreign exchange; commodity futures; and, in the case of Peters, a wide variety of financial assets.

The results from these studies have been criticized in the more recent literature.

[4] Although not of direct concern to this study, there is recent evidence suggesting the presence of long memory in asset-return volatilities. Crato and de Lima (1994) and Ding et al. (1993) report very slowly decaying rates of decay in the autocorrelation functions of the volatility in certain aggregate stock-return series. Brock and de Lima (1995) find that short-term memory is rejected by the MRS and GPH tests for a large fraction of volatility series associated with daily common-stock returns. Baillie et al. (1993) and Bollerslev and Mikkelsen (1993) respectively find evidence of long memory in squared innovations of US$/DM exchange rate and the S&P 500 stock index. This evidence is found by means of estimating fractionally-integrated GARCH models to these series. The long-memory stochastic volatility models of Harvey (1993) and Breidt et al. (1994) also yield such evidence in certain aggregate stock-return series.

This criticism focuses mainly on the difficulty of R/S analysis in distinguishing between long-memory and short-term dependent processes (cf. Davies and Harte, 1987; Wallis and Matalas, 1970) or, similarly, the lack of distribution theory by which to conduct formal hypothesis tests for long memory under relevant short-term dependence nulls (cf. Lo, 1991). Principally for this reason, R/S analysis of long memory in economic time series has largely been abandoned recently in favor of Lo's MRS test, a test which is less susceptible to this criticism.

The chief purpose of this paper is to re-examine the claims by Greene and Fielitz (1977) of widespread evidence of long memory in daily common-stock returns. Our study is based on the MRS test. We apply the MRS test to the daily return series of 1,952 ordinary common stocks and evaluate the test based on asymptotic and bootstrapped critical values. Our sample consists of all ordinary common stocks listed in the 1991 *Daily Stock Files* of the Center for Research in Securities Prices (CRSP) with at least 750 contiguous stock-return observations. Following Lo, we compute MRS test statistics corresponding to fixed autocovariance lag-truncation lengths of 90, 180, 270, and 360 periods to control for short-term dependence in the return series. We also compute test statistics corresponding to other autocovariance lag-truncation lengths which are sample-size dependent.

Our main results can be separated into two categories: results relating to the MRS test itself and results relating to the application of the test to the panel of stock-return series. Regarding this first category, we find that the test is sensitive to conditioning on the 'survival' of a series. Brown et al. (1993) show that by conditioning on the survival a firm, tests for long-range dependence (e.g., autocorrelation- and R/S-based tests) can be biased in favor of indicating the presence of such dependence in the firm's return series. Moreover, these authors further argue that bootstrapped versions of such tests should also be sensitive to conditioning on survival. We confirm their predictions by means of Monte Carlo simulations. Left-tailed asymptotic and bootstrapped MRS tests have a tendency to reject the short-term dependence null (when true) with a frequency greater than their given sizes when the tests are applied to series which are conditioned on their survival.

We also find through Monte Carlo simulations that the MRS test is sensitive to violations of its condition that fourth moments exist in the series to which it is applied. When its fourth-moment condition is not met, the test leads to left-tailed rejections of its short-term dependence null (when otherwise true) with a frequency greater than its nominal size. Alternatively, we find that a bootstrapped version of the test is insensitive to this moment-condition failure.

The chief conclusion from the second category of results is that long memory is not a widespread characteristic of common stocks. Such a conclusion is in contrast to the conclusion of Greene and Fielitz in particular and conflicts with the claims of Peters in general. We find that the proportions of left-tailed 5% significant rejections of the short-term dependence null by the MRS test, which can be indicative of antipersistent long memory, range from 2% to 6% across the tests based on selected sample-size dependent autocovariance lag-truncation lengths. The corresponding right-tailed proportions, possible indications of persistent long memory, range from 4% to 10%.

Stronger conclusions can be drawn from the rejection rates across the panel of return series, but they require caveats. Using the 2.5% and 5% MRS rejection rates and asymptotic p-values, we evaluate tests of the hypothesis that *all* of the series conform to Lo's short-term dependence null. For the asymptotic tests, there are only two cases where this hypothesis is rejected at conventional significance levels. These cases correspond to certain left-tailed rejection rates. Consequently, these excessive rejection rates might be due to the effects of moment-condition failure and survivorship bias, rather than to the presence of antipersistent long memory in a statistically significant fraction of series. On the face of this evidence one might question whether any of the return series display evidence of long memory.

On the other hand, the hypothesis that all the return series follow Lo's short-term dependence null is rejected by all tests based on selected right-tailed bootstrapped MRS tests. These excessive rates can neither be attributed to the effects of survivorship bias nor to moment-condition failures. Moreover, we find by means of estimating logit models of the event of a rejection by the asymptotic and bootstrapped tests that the returns of firms with heavy-tailed return distributions are more likely to yield right-tailed rejections by the asymptotic and bootstrapped tests, and are less likely to yield left-tailed rejections. This cannot be a manifestation of moment-condition failure; and so, we conclude that tail thickness may proxy for some other information to which the MRS test is sensitive. We also find that the right-tailed rejections are linked to firms with relatively large risk-adjusted average returns. From these results we conclude that there is evidence suggestive of persistent long memory in some of the series.

Evidence consistent with the effects of survivorship bias is also uncovered by the logit models. The returns of firms which eventually fail or are merged are more likely to generate left-tailed rejections, as are those corresponding to shorter-lived firms. These effects are statistically significant, however, only for the firms in our sample which eventually merged.

The remainder of this paper proceeds as follows. Section 2 describes the MRS test. A description of the data and testing methods along with the results of the application of the test are contained in Section 3. In Section 4 we discuss the findings of a logit study of the event of a rejection by the test. Lastly, we offer a summary in Section 5.

2. The modified R/S test

We begin with a discussion of Lo's modification of the Hurst–Mandelbrot rescaled range statistic. The discussion borrows heavily from Lo (1991, pp. 1281–1296).

2.1. The test statistic

Consider a time-series sample $\{x_t\}_{t=1}^{n}$ with sample mean $\bar{x}(n)$ and sample autocovariances $\hat{\gamma}_j(n)$, $j = 0,\ldots,q$, where q denotes the autocovariance lag-truncation length. The modified R/S (or, MRS) test statistic is given by

$$Q(n,q) = \left[n \cdot \hat{\sigma}^2(n,q)\right]^{-1/2}\left[\max_{1\le k\le n}\sum_{j=1}^{k}\left[x_j - \bar{x}(n)\right] - \min_{1\le k\le n}\sum_{j=1}^{k} \times\left[x_j - \bar{x}(n)\right]\right], \tag{1}$$

where

$$\hat{\sigma}^2(n,q) = \hat{\gamma}_0(n) + \sum_{j=1}^{q}\omega_j(q)\cdot\hat{\gamma}_j(n), \quad \omega_j(q) = 1 - \frac{j}{q+1}, \; q < n. \tag{2}$$

The bracketed term in Eq. (1) is the range (i.e., the maximum minus the minimum over k) of the partial sums of the first k deviations of x_j from its sample mean. This range statistic is scaled by dividing the range by the square root of $\hat{\sigma}^2(n, q)$ in Eq. (2), which is an estimator of the variance of the partial sums. The difference between the Hurst–Mandelbrot R/S and the MRS statistics centers on the variance estimator. Unlike the Hurst–Mandelbrot variance estimator, the variance estimator in the MRS statistic includes autocovariance terms which in general are needed if the process which generates $\{x_t\}$ is short-range dependent. [5]

2.2. The asymptotic distribution under Lo's short-term dependence null

Lo derives the asymptotic distribution of the MRS test statistic under a certain short-term dependence null. This null hypothesis concerns a stochastic process, $X_t \equiv \mu + \epsilon_t$, where μ denotes an arbitrary but fixed parameter and ϵ_t denotes a zero-mean random variable. The process $\{X_t\}$ is assumed to be strong-mixing. Strong-mixing is one way of measuring the degree of temporal dependence in a stochastic process. It is defined in terms of α-mixing coefficients, α_k, given by

$$\alpha_k \equiv \sup_{j}\;\sup_{\{A\in\mathscr{A}_{-\infty}^{j},\, B\in\mathscr{A}_{j+k}^{\infty}\}} |\mathrm{Prob}(A\cap B) - \mathrm{Prob}(A)\cdot\mathrm{Prob}(B)|, \tag{3}$$

where $\mathscr{A}_t^s$ is the Borel σ-field generated by $\{X_t,\ldots,X_s\}$. The time-series process $\{X_t\}$ is said to be strong-mixing if $\lim_{k\to\infty}\alpha_k = 0$. As can be seen from the definition of the α-mixing coefficients, strong-mixing implies a form of asymptotic independence. In effect, strong-mixing requires that the maximal dependence

[5] The weights $\omega_j(q)$ in (2) are Bartlett weights. Alternative weighting schemes and optimal selection criteria for q for covariance-matrix estimation are discussed in Andrews (1991).

between two events becomes trivially small as their period of separation increases without bound.

In order to derive the asymptotic distribution of the MRS test statistic for strong-mixing processes where some form of the law of large numbers and the functional central limit theorem can be applied, some other conditions must be placed on $\{X_t\}$. These conditions place restrictions on the maximal moments, the degree of distributional heterogeneity, and the maximal degree of dependence in $\{X_t\}$. Moreover, some other conditions are needed to ensure consistency of $\hat{\sigma}^2(n, q)$ in Eq. (2). In order for the law of large numbers and the functional central limit to be obtained, Lo imposes the following three conditions:

$$\sup_t \mathrm{E}\left(|\epsilon_t|^{\beta}\right) < \infty, \text{ for some } \beta > 2, \tag{4}$$

$$0 < \sigma^2 = \lim_{n \to \infty} \mathrm{E}\left[(1/n)\left(\sum_{j=1}^{n} \epsilon_j\right)^2\right] < \infty, \tag{5}$$

and $\{\epsilon_t\}$ is strong-mixing with α-mixing coefficients that satisfy

$$\sum_{k=1}^{\infty} \alpha_k^{1-(2/\beta)} < \infty. \tag{6}$$

The restriction in Eq. (4) requires that the second moments of the ϵ_t's are finite; Eq. (5) restricts the degree of their distributional heterogeneity; and Eq. (6) combined with Eq. (4) governs the rate of decay of the mixing coefficients. In addition, if it is also assumed that as n increases without bound, q also increases without bound such that $q \sim o(n^{1/4})$, and if the maximal moment condition in Eq. (4) is further restricted to fourth moments, namely

$$\sup_t \mathrm{E}\left(|\epsilon_t|^{\beta}\right) < \infty, \quad \text{for some } \beta > 4, \tag{4'}$$

then consistency of $\hat{\sigma}^2(n, q)$ is ensured. This result follows from Theorem 4.2 of Phillips (1987). It should also be pointed out for future reference that Andrews (1991) improves the rate restriction of $q \sim o(n^{1/4})$ to $o(n^{1/2})$.

Under the assumption that $\{X_t\}$ is short-term dependent satisfying the conditions above, Lo shows that the asymptotic cumulative distribution function $F_V(v)$ of $V \equiv Q(n, q)$ is given by

$$F_V(v) = 1 + 2\sum_{k=1}^{\infty} (1 - 4k^2v^2)\mathrm{e}^{-2(kv)^2}. \tag{7}$$

Critical values of $F_V(v)$ are 0.809, 0.861, 1.747 and 1.862 at the 2.5%, 5.0%, 95.0% and 97.5% probability levels.

2.3. Long-range dependent alternatives

Lo designs his short-term dependence null in order to distinguish between weakly-dependent (i.e., mixing) time-series processes and strongly-dependent processes such as fractionally-integrated ARMA processes. As mentioned in the introduction, these strongly-dependent processes are not strong-mixing and have autocorrelation functions which decay at rates much slower than those of weakly-dependent processes. Lo shows that the MRS test is a consistent test against a class of long-range dependent stationary alternatives. This class includes all fractionally-differenced Gaussian ARMA models with fractional difference parameters, d, of $d \in (-0.5, 0.5)$.

Fractionally-differenced Gaussian ARMA (ARFIMA) specifications with $d \in (-0.5, 0.5)$ can be expressed as

$$\phi(B)\nabla^d(X_t - \mu) = \theta(B)Z_t, \quad d \in (-0.5, 0.5), \{Z_t\} \sim i.i.d.\mathrm{N}(O, \sigma^2),$$
$$BX_t \equiv X_{t-1},$$
$$\nabla^d \equiv (1-B)^d = \sum_{j=0}^{\infty} \pi_j B^j, \quad \pi_j = \prod_{0<k\le j}[(k-1-d)/k], \tag{8}$$

where $\phi(B)$ and $\theta(B)$ respectively denote autoregressive and moving-average lag polynomials. Theorem 3.3 of Lo (1991) shows that the MRS test statistic converges in probability to zero for these antipersistent long-memory processes (i.e., for $d < 0$); while for these persistent long-memory processes (where $d > 0$), the test diverges in probability to infinity. As a result, the tails of the MRS test are able to distinguish between these differing types of long-range behavior.

2.4. Other alternatives

While Lo's short-term dependence null is specifically designed to distinguish weakly-dependent processes from strongly-dependent processes, there are strong-mixing processes, however, which violate the assumptions of Lo's short-term dependence null. For example, a strong-mixing process with a maximal moment less than 4 violates the moment condition in Eq. (4′); the first difference of a stationary process will violate the heterogeneity condition in Eq. (5), since its spectral density at frequency zero vanishes; a strong-mixing process with a non-constant μ violates the short-term dependence null; and Lo (1991, p. 1283) provides other strong-mixing processes which violate Eq. (5). On the other hand, Lo's short-term dependence null covers a wide variety of conventional time-series processes, including Gaussian finite-order stationary ARMA processes, certain heterogeneously-distributed processes, as well as many of the stochastic models of persistence such as those of Fama and French (1988) and Poterba and Summers (1988).

2.5. Some finite-sample properties

Table 1 displays results of a few Monte Carlo experiments relating to the size and power of asymptotic and bootstrapped MRS tests. The experiments corresponding to the results in Panels A–D are similar to Lo's (1991, Tbls. 5 and 6) size and power experiments. Panels A and B correspond to Gaussian ARFIMA processes with AR and MA lag-truncation lengths of zero and with fractional-difference parameters of $d = 1/3$ and $d = -1/3$ respectively. Panel C corresponds to an i.i.d. N(0, 1) process; and Panel D corresponds to an AR(1) process with an autocorrelation coefficient of $\rho = 0.5$ and with i.i.d. normal innovations. [6] Table 1 also includes the results of two size experiments in Panels C′ and D′ corresponding to experiments where the i.i.d. normal innovations of experiments C and D are replaced with symmetric-about-zero i.i.d. Pareto innovations with a maximal moment of $\beta = 2.7$. [7] These experiments are designed to evaluate the sensitivity of the MRS test to departures from its fourth-moment condition (or, hereafter FMC) in Eq. (4′). The variances of the innovations in the processes corresponding to all of the above experiments are selected so that the processes possess a unit variance. Lastly, Panel E displays the results of an experiment where an independently-distributed N(μ, 1) process undergoes a sequence of $m = 4$ alternating shifts in its mean, μ. Such a process violates Lo's short-term dependence null even though it is independently distributed. This experiment is similar in nature to one in Cheung (1993, Sctn. 3.3).

The sample sizes considered in the experiments are $n = 750$ and 1500. Tests based on q-lengths $q(n)$ of (int)($n^{1/4}$), (int)($n^{1/3}$), (int)($n^{1/2}$), and lengths based on a data-dependent formula of Andrews (1991), $q^*(n)$, are evaluated in each experiment. The q-length, $q^*(n)$, is given by

$$q^*(n) = (\mathrm{int})\left[\left(\frac{3n}{2}\right)^{1/3} \cdot \left(\frac{2\hat{\rho}}{1-\hat{\rho}^2}\right)^{2/3}\right], \tag{9}$$

where $\hat{\rho}$ denotes the first-order sample autocorrelation coefficient associated with $\{x_t\}$. [8] The size and power results relating to the asymptotic tests are based on 10,000 replications. The results relating to the bootstrapped tests are based on

[6] Realizations of the random variables used in Monte Carlo simulations throughout this study are generated with the Box–Muller method and a combination of three linear congruential pseudo-random number generators. See Press et al. (1988, pp. 204–217).

[7] The choice of a maximal moment of $\beta = 2.7$ stems from results presented in Section 4. This value of β is the average Loretan and Phillips (1994) right-tailed maximal-moment estimate across the return series.

[8] This formula comes from equations (6.2) and (6.4) in Andrews (1991). Andrews shows that this q-length formula has optimal properties for Bartlett weights when the underlying data is generated by an AR(1) process. Optimal q-length formulas for other weighting schemes and data-generating processes can also be found in Andrews (1991). Throughout the remainder of the text we will ignore the integer cast, (int), on the q-lengths.

Table 1
2.5% and 5% rejection rates of asymptotic and bootstrapped MRS tests against seven processes[a]

		Left tail				Right tail			
		2.5%		5%		2.5%		5%	
q	n	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP
Panel A: $(1-L)^d X_t = \epsilon_t$, $\epsilon_t \sim$ i.i.d. $N(0, \sigma_\epsilon^2)$, $\sigma_\epsilon^2 = \Gamma^2(1-d)/\Gamma(1-2d)$, $d = 1/3$									
$q^*(n)$	750	0.000	0.001	0.001	0.002	0.603*	0.681*	0.504*	0.596*
	1500	0.000	0.000	0.000	0.000	0.774*	0.812*	0.699*	0.754*
$n^{1/4}$	750	0.000	0.000	0.000	0.000	0.782*	0.830*	0.714*	0.758*
	1500	0.000	0.000	0.000	0.000	0.907*	0.939*	0.870*	0.905*
$n^{1/3}$	750	0.000	0.000	0.000	0.001	0.678*	0.735*	0.589*	0.663*
	1500	0.000	0.000	0.000	0.000	0.817*	0.857*	0.754*	0.798*
$n^{1/2}$	750	0.003	0.006	0.006	0.009	0.246*	0.384*	0.144*	0.293*
	1500	0.001	0.003	0.003	0.006	0.443*	0.508*	0.331*	0.410*
Panel B: $(1-L)^d X_t = \epsilon_t$, $\epsilon_t \sim$ i.i.d. $N(0, \sigma_\epsilon^2)$, $d = -1/3$									
$q^*(n)$	750	0.811*	0.789*	0.904*	0.873*	0.000	0.000	0.000	0.000
	1500	0.956*	0.956*	0.983*	0.984*	0.000	0.000	0.000	0.000
$n^{1/4}$	750	0.886*	0.854*	0.947*	0.921*	0.000	0.000	0.000	0.000
	1500	0.985*	0.980*	0.995*	0.992*	0.000	0.000	0.000	0.000
$n^{1/3}$	750	0.712*	0.685*	0.834*	0.816*	0.000	0.000	0.000	0.000
	1500	0.897*	0.894*	0.953*	0.954*	0.000	0.000	0.000	0.000
$n^{1/2}$	750	0.026	0.087*	0.080*	0.159*	0.000	0.000	0.000	0.000
	1500	0.101*	0.168*	0.223*	0.310*	0.000	0.000	0.000	0.000
Panel C: $X_t \sim$ i.i.d. $N(0,1)$									
$q^*(n)$	750	0.010	0.032	0.027	0.052	0.019	0.042	0.005	0.022
	1500	0.037*	0.030	0.069*	0.051	0.045	0.064	0.023	0.034
$n^{1/4}$	750	0.042*	0.032	0.079*	0.052	0.039	0.042	0.020	0.024
	1500	0.034*	0.032	0.065*	0.051	0.044	0.064	0.020	0.038
$n^{1/3}$	750	0.035*	0.027	0.068	0.061	0.036	0.044	0.017	0.02
	1500	0.030*	0.031	0.060*	0.050	0.042	0.062	0.019	0.04
$n^{1/2}$	750	0.030*	0.032	0.061*	0.052	0.033	0.038	0.015	0.01
	1500	0.014	0.032	0.035	0.062	0.031	0.067*	0.011	0.03
Panel C′: $X_t \sim$ i.i.d. symmetric Pareto, $E(X_t) = 0$, $E(X_t^2) = 1$, $\beta = \underset{m}{sup} E\|X_t^m\| = 2.7$									
$q^*(n)$	750	0.047*	0.017	0.088*	0.041	0.017	0.038	0.006	0.015
	1500	0.048*	0.027	0.084*	0.052	0.020	0.043	0.009	0.02
$n^{1/4}$	750	0.040*	0.018	0.080*	0.053	0.019	0.040	0.009	0.01
	1500	0.043*	0.026	0.082	0.053	0.019	0.040	0.009	0.01
$n^{1/3}$	750	0.035*	0.019	0.074*	0.047	0.013	0.039	0.004	0.01
	1500	0.040*	0.023	0.078*	0.055	0.019	0.041	0.007	0.01
$n^{1/2}$	750	0.016	0.020	0.042	0.046	0.007	0.046	0.001	0.01
	1500	0.022	0.033	0.053	0.056	0.013	0.030	0.004	0.01

Table 1 (continued)

		Left tail				Right tail			
		2.5%		5%		2.5%		5%	
q	n	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP
Panel D: $X_t = \rho X_{t-1} + \epsilon_t$, $\rho = 0.5$, $\epsilon_t \sim$ i.i.d. $N(0, [1-\rho^2])$									
$q^*(n)$	750	0.027	0.027	0.052	0.057	0.036	0.070*	0.015	0.03
	1500	0.029*	0.032	0.057*	0.064	0.053	0.068*	0.026	0.03
$n^{1/4}$	750	0.020	0.015	0.039	0.031	0.071*	0.126	0.038	0.07*
	1500	0.021	0.017	0.042	0.038	0.084*	0.104*	0.049*	0.06*
$n^{1/3}$	750	0.025	0.023	0.049	0.046	0.048	0.101*	0.024	0.04*
	1500	0.029*	0.027	0.056*	0.054	0.060*	0.079*	0.032*	0.04*
$n^{1/2}$	750	0.017	0.036	0.040	0.069*	0.015	0.045	0.004	0.02
	500	0.020	0.042*	0.045	0.074*	0.033	0.049	0.011	0.02
Panel D′: $X_t = \rho X_{t-1} + \epsilon_t$, $\rho = 0.5$, $\epsilon_t \sim$ i.i.d. symmetric Pareto, $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = (1-\rho^2)$, $\beta = 2.7$									
$q^*(n)$	750	0.029*	0.025	0.060*	0.051	0.017	0.056	0.006	0.028
	1500	0.031*	0.022	0.064*	0.051	0.024	0.040	0.010	0.017
$n^{1/4}$	750	0.019	0.013	0.039	0.027	0.038	0.091*	0.018	0.057*
	1500	0.020	0.006	0.041	0.020	0.046	0.074*	0.023	0.033
$n^{1/3}$	750	0.025	0.019	0.052	0.042	0.025	0.069*	0.010	0.038*
	1500	0.028	0.016	0.058*	0.043	0.030	0.046	0.013	0.022
$n^{1/2}$	750	0.022	0.039*	0.052	0.066*	0.007	0.047	0.011	0.021
	1500	0.027	0.040*	0.059*	0.074*	0.013	0.030	0.004	0.013
Panel E: $X_t = 0.1 \cdot (-1)^g + \epsilon_t$, $\epsilon_t \sim$ i.i.d. $N(0,1)$, $g = (\text{int})[(m+1)t/n]$, $m = 4$									
$q^*(n)$	750	0.004	0.002	0.010	0.004	0.164*	0.220*	0.098*	0.139*
	1500	0.000	0.000	0.000	0.000	0.385	0.424*	0.271*	0.324*
$n^{1/4}$	750	0.003	0.002	0.009	0.003	0.138*	0.200*	0.076*	0.123*
	1500	0.000	0.000	0.000	0.000	0.348*	0.397*	0.233*	0.286*
$n^{1/3}$	750	0.001	0.001	0.007	0.055	0.117*	0.181*	0.062*	0.107*
	1500	0.000	0.000	0.000	0.000	0.302*	0.361	0.199*	0.254*
$n^{1/2}$	750	0.001	0.005	0.005	0.014	0.042	0.108*	0.014	0.052*
	1500	0.000	0.000	0.000	0.002	0.139*	0.205*	0.069*	0.126*

[a] Asterisks denote 1% significant rejections of the hypothesis that the rejection rate is less than or equal to its corresponding nominal size.

1,000 replications where for each replication the test statistics corresponding to the realized series of the replication are evaluated on the basis of critical values stemming from 1,000 time-scrambled shufflings of the realized series. Finally, we test whether the rejection rates $R(s)$ based on N replications are greater than the nominal sizes s. We evaluate such tests by means of the statistic, $N^{1/2}[R(s) - s]/[s(1-s)]^{1/2}$, which is asymptotically distributed $N(0,1)$ as $N \to \infty$ under the assumption that $N \cdot R(s)$ is distributed binomial (N, s).

The results reported in Panels A and B relating to the persistent and antipersistent Gaussian fractionally-differenced processes indicate that the tests have relatively high power in rejecting the short-term dependence null when they are based on q-lengths of $q \leq n^{1/3}$. The asymptotic and bootstrapped tests yield similar power values and are comparable to the results of Lo (1991).

In regard to the size of the tests, the results in Panel C indicate that the asymptotic test has a slight tendency to generate too many left-tailed rejections for the i.i.d. normal case. The results in Panel C′, corresponding to the experiment where the tests are applied to i.i.d. symmetric Pareto realizations, indicate that the asymptotic tests based on $q \leq n^{1/3}$ are sensitive to FMC failures. In these cases there is a shift to the left of the finite-sample distribution relative to the asymptotic distribution. The result is an over-abundance of left-tailed rejections which does not appear to decline with increasing n. Similar findings are reported in Brock and de Lima (1995, Sctn. 3.1). Panels D and D′ indicate that the bootstrapped test can be sensitive to its failure to incorporate short-term dependence into its null distribution. This is especially apparent from Panel D for the right-tailed tests based on $q \leq n^{1/3}$. It should be noted, however, that this sensitivity generally declines with increasing n. And lastly, the results in Panel E relating to the effects of mean shifts on the MRS test indicate, as in Cheung (1993), that the MRS test is very sensitive to such effects. Over-rejections occur in the right-tailed tests, and the bootstrapped tests appear somewhat more sensitive to shifts in mean.

We conduct one other set of experiments. As mentioned in the introduction, Brown et al. (1993) argue that tests for long-range dependence can be sensitive to conditioning on survival. Table 2 displays results of a set of experiments in which the MRS tests are applied to the realized return series of an i.i.d. N(0, σ^2) returns process, say $\{r_t\}$, provided that its corresponding price series, $\{p_t\}$, given by

$$p_t \equiv 1 + \sum_{\tau=1}^{t} r_\tau, \tag{10}$$

remains above some threshold, k, throughout the horizon, $t = 1,\ldots,n$. Following these authors, we set $\sigma^2 = 0.04$, and evaluate the finite-sample size of the MRS tests for the cases where $k = (0.095,\ 0.182,\ 0.262,\ 0.336,$ and $0.405)$. All other aspects of these experiments are the same as those relating to Table 1. As shown in the table, the left-tailed asymptotic tests based on $q \leq n^{1/3}$ all yield rejection rates which are significantly larger than their sizes. These excessive rates decline with increasing n, but not considerably. Many of the bootstrapped tests also yield excessive left-tailed rejections, but the excessive rates are lower than those corresponding to the asymptotic tests.

Overall, the results from these experiments suggest the possibility that right-tailed rejections by asymptotic MRS tests might simply be the result of size distortions arising from shifts in means (Table 1, Panel E) or from slow-convergence problems (Panel D). Right-tailed rejections by bootstrapped tests might be the result of shifts in means (Panel E) or from failure to account for short-term dependence in the bootstrap (Panels D and D′). Alternatively, left-tailed rejections by asymptotic tests might be the result of slow-convergence problems (Panel C),

Table 2
2.5% and 5% rejection rates of asymptotic and bootstrapped MRS tests against the process $\{r_t\}$ defined below.[a]
$\{r_t\} \sim$ i.i.d. N(0,0.04) where $p_t \equiv 1+\Sigma_{\tau=1}^{t} r_t \geq k$ for all t.

			Left tail				Right tail			
			2.5%		5%		2.5%		5%	
k	q	n	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP	ASMP	BTSP
0.095	$q^*(n)$	750	0.058*	0.038*	0.100*	0.073*	0.022	0.040	0.009	0.021
		1500	0.053*	0.042*	0.094*	0.074*	0.033	0.035	0.018	0.021
	$n^{1/4}$	750	0.048*	0.036	0.089*	0.066*	0.019	0.039	0.007	0.021
		1500	0.047*	0.042*	0.086*	0.077*	0.031	0.037	0.017	0.020
	$n^{1/3}$	750	0.041*	0.031	0.080	0.067	0.018	0.037	0.006	0.019
		1500	0.043*	0.040*	0.078*	0.071*	0.030	0.035	0.015	0.020
	$n^{1/2}$	750	0.013	0.033	0.037	0.053	0.009	0.048	0.02	0.025
		1500	0.021	0.035	0.047	0.068*	0.022	0.042	0.009	0.023
0.182	$q^*(n)$	750	0.056*	0.035	0.100*	0.071*	0.022	0.041	0.009	0.021
		1500	0.048*	0.038*	0.091*	0.069*	0.031	0.035	0.016	0.021
	$n^{1/4}$	750	0.048*	0.034	0.088*	0.064*	0.020	0.040	0.007	0.022
		1500	0.043*	0.037*	0.083*	0.073*	0.030	0.037	0.015	0.020
	$n^{1/3}$	750	0.041*	0.029	0.079*	0.064	0.018	0.038	0.006	0.020
		1500	0.039*	0.035	0.075*	0.066*	0.029	0.036	0.044	0.020
	$n^{1/2}$	750	0.014	0.027	0.036	0.049	0.010	0.052	0.002	0.027
		1500	0.019	0.033	0.044	0.065	0.021	0.043	0.008	0.024
0.262	$q^*(n)$	750	0.055*	0.037*	0.099*	0.070*	0.023	0.044	0.009	0.021
		1500	0.049*	0.041*	0.088*	0.070*	0.031	0.038	0.017	0.022
	$n^{1/4}$	750	0.045*	0.036	0.084*	0.063	0.021	0.042	0.007	0.022
		1500	0.043*	0.035*	0.080*	0.073*	0.030	0.039	0.015	0.021
	$n^{1/3}$	750	0.039*	0.031	0.076	0.066*	0.018	0.041	0.007	0.020
		1500	0.038	0.038*	0.074*	0.067*	0.029	0.038	0.014	0.022
	$n^{1/2}$	750	0.013	0.030	0.034	0.053	0.012	0.056	0.002	0.028
		1500	0.018	0.036	0.044	0.068*	0.022	0.046	0.008	0.026
0.336	$q^*(n)$	750	0.052	0.036	0.095*	0.071*	0.023	0.044	0.009	0.021
		1500	0.048*	0.042*	0.086*	0.072*	0.032	0.041	0.046	0.027
	$n^{1/4}$	750	0.043*	0.035	0.081*	0.063	0.020	0.040	0.007	0.021
		1500	0.040*	0.039*	0.078*	0.074*	0.030	0.042	0.015	0.025
	$n^{1/3}$	750	0.037*	0.029	0.072*	0.064	0.019	0.054	0.006	0.021
		1500	0.037*	0.040*	0.070*	0.069*	0.029	0.040	0.014	0.024
	$n^{1/2}$	750	0.013	0.029	0.034	0.053	0.010	0.044	0.002	0.030
		1500	0.018	0.038*	0.041	0.073*	0.021	0.043	0.008	0.023
0.405	$q^*(n)$	750	0.053*	0.039*	0.092*	0.071*	0.025	0.048	0.009	0.023
		1500	0.048*	0.038*	0.087*	0.062	0.032	0.046	0.016	0.028
	$n^{1/4}$	750	0.045*	0.038*	0.080*	0.061	0.021	0.048	0.007	0.023
		1500	0.043*	0.035	0.079*	0.063	0.030	0.044	0.015	0.028
	$n^{1/3}$	750	0.037*	0.029	0.073*	0.065	0.019	0.046	0.06	0.023
		1500	0.039*	0.036	0.072*	0.062	0.029	0.043	0.014	0.026
	$n^{1/2}$	750	0.013	0.029	0.034	0.055	0.010	0.052	0.002	0.031
		1500	0.018	0.035	0.043	0.067*	0.022	0.046	0.008	0.027

[a] Asterisks denote 1% significant rejections of the hypothesis that the rejection rate is less than or equal to its corresponding nominal size.

FMC failures (Panel C′), or from survivorship bias (Table 2). And finally, left-tailed rejections by bootstrapped tests might reflect survivorship bias, as indicated in Table 2.

3. An application to common stock returns

3.1. The returns series

The daily common stock returns in our sample are taken from the 1991 CRSP *Daily Stock Files*. These files provide information on stocks listed on the New York and American Stock Exchanges over the period July 2, 1962 to December 31, 1991. The longest possible trading history for a stock is 7,421 trading days. Only those ordinary common stocks for which there are 750 or more contiguous stock-return observations are included in the sample. There are $N = 1952$ such stocks which satisfy this criterion in the 1991 CRSP files.

For future reference, let $t = 1$ (July 3, 1962),...,$t = 7420$ (December 31, 1991) denote the days over which the sample of stock-return series are observed. Let $n_i \geq 750$ denote the length of the return series corresponding to the ith firm in the sample. Also, let $T_i = \{\tau_i(1),\ldots,\tau_i(n_i)\}$ denote the set of days over which returns are observed for the ith firm. The return series are denoted by $\{r_{i,t}\}$, $t \in T_i$, where $i = 1,\ldots,N$ and $1 \leq \tau_i(1) < \tau_i(n_i \geq 750) \leq 7420$. For the sample of stocks, the average and standard deviation for the number of contiguous return observations are respectively 3,173 and 2,344.

The stock-return series correspond to the CRSP holding period returns. (See the 1992 *Stock File Guide* (CRSP, 1992, p. 30).) Returns, adjusted for both dividends and stock splits, are calculated as

$$r_{i,t} = \frac{p_{i,t} \cdot f_{i,t} + d_{i,t}}{p_{i,t}} - 1, \tag{11}$$

for all firms $i = 1,\ldots,N$ and times $t = 1,2,\ldots,n_i$. In Eq. (11), $p_{i,t}$ denotes the last sale price or closing bid/ask average at time t, $d_{i,t}$ denotes a cash adjustment at time t, $f_{i,t}$ denotes a price adjustment factor at time t, and t' denotes the time of the last sale price or closing bid/ask average at the last available price (provided that it is no more than ten trading days prior to period t).

3.2. Testing methods

We apply the MRS test to the sample of stock-return series using a variety of q-lengths. For each firm i, we select lengths of $q_i = n_i^{1/4}$, $n_i^{1/3}$, $n_i^{1/2}$, and $q_i = q^*(n_i)$. And, following Lo (1991) in his application to daily and weekly aggregate stock-return series, we select fixed q-lengths of $q_i = q = 90$, 180, 270, and 360 trading periods.

We then compare the test statistics corresponding to each of the q-lengths for each of the series to the 2.5%, 5%, 95%, and the 97.5% asymptotic critical values under Lo's short-term dependence null. In addition, we compare the test statistics to bootstrapped 2.5%, 5%, 95%, and the 97.5% critical values for each firm. The bootstrapped critical values for the return series of a firm are based on the test statistics stemming from 1,000 time-scrambled shufflings of its return series.

This bootstrap procedure, where the elements of the series are time-scrambled and are thereby independent, is consistent with the null of strong-mixing with α-mixing coefficients satisfying Eq. (6). On the other hand, if the unscrambled series violates the moment condition in Eq. (4′), then its time-scrambled counterparts will violate this condition as well, since time-scrambling preserves the unconditional properties of the original series. (Depending on its nature, a violation of the heterogeneity condition in Eq. (5) may or may not be preserved by time scrambling.) As a result, when using such bootstrapped critical values to evaluate a test statistic, the test is robust to violations of the moment condition and certain violations of the heterogeneity condition of the short-term dependence null, while being sensitive to departures from the strong-mixing assumption (cf. Table 1, Panels A, B, and C′). This is not necessarily an undesirable property, given that in Lo's framework strong-mixing is the salient feature distinguishing short- from long-range dependence. [9]

Alternatively, we should emphasize that our bootstrap procedure based on time scrambling necessarily omits any short-term dependence present in a series. Designing and applying alternative bootstraps which include short-term dependence is well beyond the scope of this study. Nonetheless, we recognize that the nature of our bootstrap procedure could lead to size distortions in the test statistics (cf. Table 1, Panels D and D′); and so, we exercise some caution in their interpretation.

3.3. Results

Figs. 1 and 2 display the asymptotic and bootstrapped 5% and 95% critical values of the MRS test corresponding to the sample of stock-return series in relation to the n and selected q values. [10] Fig. 1 displays the left-tailed 5% critical values, while Fig. 2 displays those of the right tail.

[9] It is also worthwhile to point out that time-scrambled series from stationary Gaussian fractionally-differenced ARMA processes satisfy all the conditions of Lo's short-term dependence null. In particular, the strong-mixing and heterogeneity-condition violations of such processes are nor preserved by time scrambling.

[10] Figures displaying the left- and right-tailed 2.5% critical values are similar in nature: as a result, they are not shown here. In addition, the bootstrapped critical values corresponding to the $q^*(n_i)$ cases are also not shown.

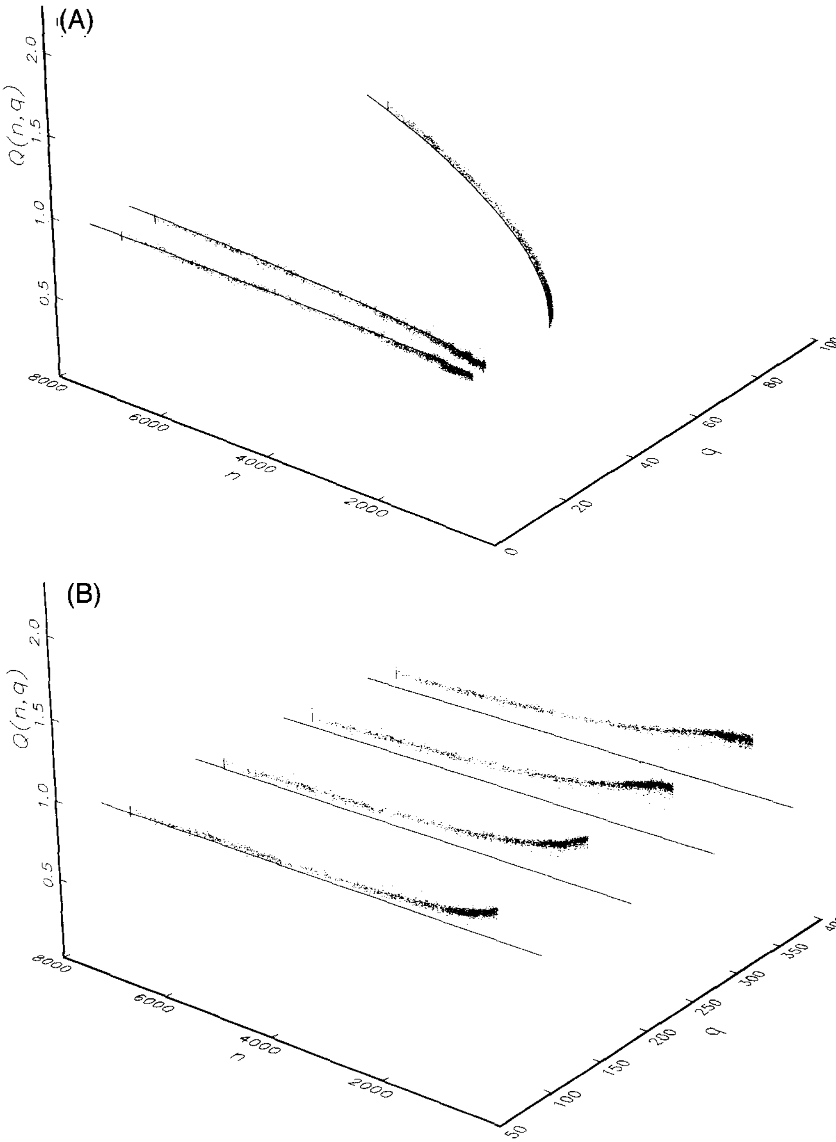


Fig. 1. Bootstrapped 5% critical values of the MRS test statistic $Q(n, q)$.

(A). For q-lengths of $n^{1/4}$, $n^{1/3}$, and $n^{1/2}$ corresponding to the panel of return series. The curves along the (n, q) plane correspond to the 5% asymptotic critical value of 0.861. Looking from left to right, the plots correspond to $q = n^{1/4}$, $n^{1/3}$, and $n^{1/2}$.

(B). For q-lengths of $q = 90$, 180, 270, 360. The lines along the (n, q) plane correspond to the 5% asymptotic critical value of 0.861.

Note in Fig. 1.A that the left-tailed bootstrapped critical values are generally smaller than the asymptotic critical value of 0.861 for the q-lengths of $n_i^{1/4}$ and $n_i^{1/3}$, and are generally larger than the asymptotic critical value for the q-lengths of $n_i^{1/2}$. On the other hand, for the fixed q-lengths of 90, 180, 270, and 360 periods, the bootstrapped critical values shown in Fig. 1.B are generally greater than the asymptotic critical value, and the difference between the bootstrapped critical values and the asymptotic value increases with increasing q and decreasing n.

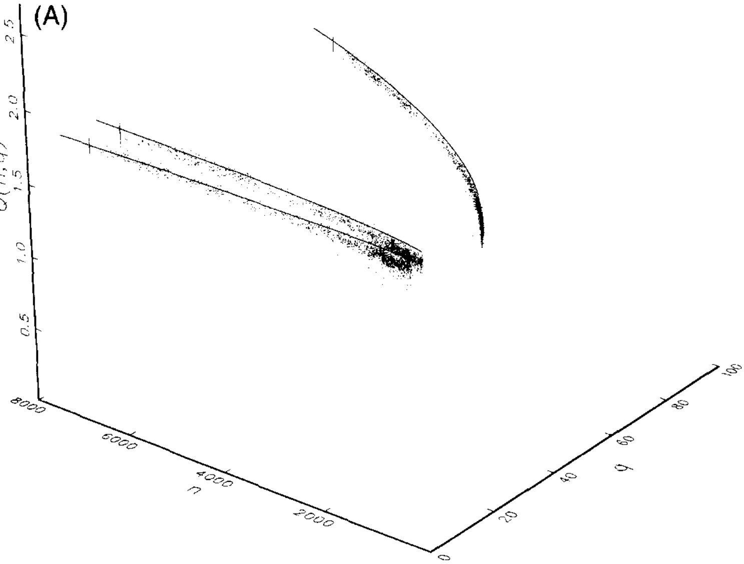
The results in Fig. 2.A indicate that the bootstrapped right-tailed 5% critical values generally lie below the asymptotic critical value of 1.747 for sample-size dependent q-lengths. Whereas, for the fixed q-lengths excluding $q = 90$, the bootstrapped critical values shown in Fig. 2.B are well above the asymptotic critical value for the relatively small sample sizes. And, with increasing q and decreasing n, the bootstrapped critical values diverge from the asymptotic value. Alternatively, for the relatively larger sample sizes, the bootstrapped critical values are below the asymptotic value, but the difference between the asymptotic and bootstrapped values declines with increasing sample sizes.

Overall, the results relating to the tests based on the sample-size dependent q-lengths of $q_i(n_i) < n_i^{1/2}$ indicate a shift-to-the-left effect of the bootstrapped distribution relative to that of the asymptotic – an effect which could be indicative of survivorship bias (cf. Section 2.5). Alternatively, the results corresponding to the fixed q-lengths indicate that the MRS test is indeed sensitive to the selection of the autocovariance lag-truncation length.

Table 3 reports the fractions of return series which are rejected at the left- and right-tailed 2.5% and 5% asymptotic and bootstrapped significance levels for each of the autocovariance lag-truncation lengths used in applying the test. Our conclusions will be drawn from the 5%-significant rejection rates corresponding to the sample-size dependent q-lengths, where the discrepancies between the asymptotic and bootstrapped critical values are generally small relative to those relating to the fixed q-lengths. [11] These results indicate that the left-tail rejections range from 2 to 6 percent, while right-tail rejections range from 4 to 10 percent.

If all the return series in our sample conform to Lo's short-term dependence null, we would expect roughly 5% of the return series to generate rejections by the test at 5% significance. By treating the event of a rejection by the test as an independent binomial random variable, we would expect the difference between the 5% rejection rate and the significance level of 0.05 to be approximately normal with a mean of zero and a standard deviation of 0.005. At 1% significance, only the rejection rates corresponding to the bootstrapped 95% critical values associated with the sample-size dependent q-lengths of $q_i(n_i)$ equal to $n_i^{1/3}$ and $n_i^{1/2}$ indicate an over-abundance of rejections.

[11] Throughout the remainder of the paper we will ignore the results of the tests bases on fixed q-lengths.


(A)
Q(n,q)
2.5
2.0
1.5
1.0
0.5
8000
6000
4000
n
2000
0
20
40
60
80
100
q

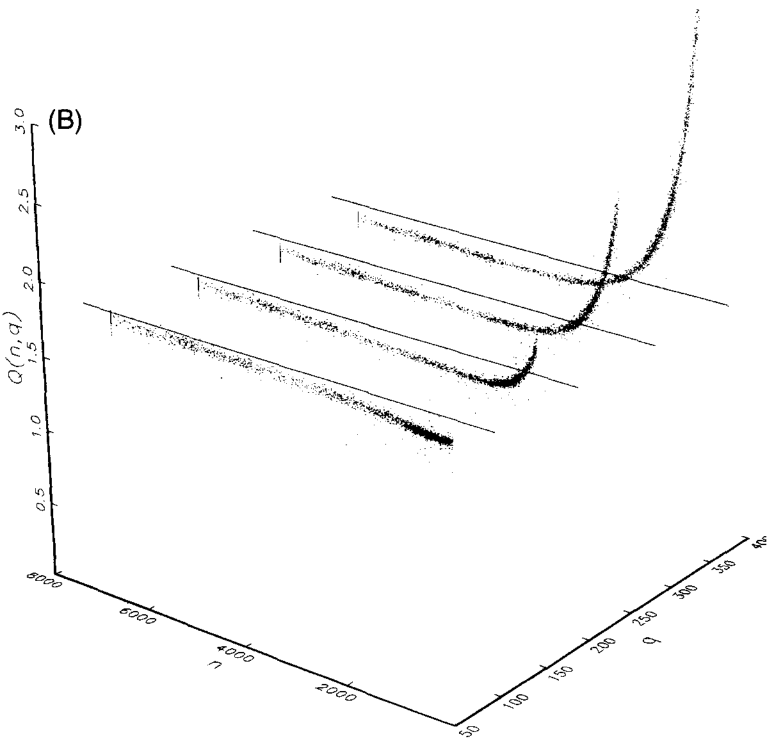

(B)
Q(n,q)
3.0
2.5
2.0
1.5
1.0
0.5
8000
6000
4000
n
2000
50
100
150
200
250
300
350
q

Table 3
Fractions of return series for which asymptotic and bootstrapped MRS test statistics yield 2.5% and 5% rejections[a]

	Left tail					Right tail				
		$R(0.025)$		$R(0.05)$			$R(0.05)$		$R(0.025)$	
q	BHPV	ASMP	BTSP	ASMP	BTSP	BHPV	ASMP	BTSP	ASMP	BTSP
$q^*(n)$	(0.00)	0.035*	0.026	0.060†	0.049	(0.99)	0.045	0.058†	0.021	0.036*
$n^{1/4}$	(0.19)	0.027	0.021	0.051	0.039	(0.99)	0.043	0.060†	0.022	0.034*
$n^{1/3}$	(0.58)	0.018	0.014	0.041	0.036	(0.99)	0.050	0.072*	0.026	0.039*
$n^{1/2}$	(0.99)	0.007	0.001	0.021	0.026	(0.99)	0.057	0.098*	0.022	0.054*
90	(0.99)	0.004	0.012	0.009	0.023	(0.99)	0.050	0.111*	0.018	0.062*
180	(0.99)	0.002	0.010	0.002	0.021	(0.20)	0.079*	0.127*	0.035*	0.076*
270	(1.0)	0.001	0.013	0.004	0.026	(0.00)	0.134*	0.132	0.085*	0.071*
360	(1.0)	0.000	0.018	0.001	0.035	(0.00)	0.191*	0.092*	0.135*	0.052*

[a] Under the assumption that the test statistics are independent across the stocks, the asterisks and daggers respectively denote 1 and 5 percent significant right-tailed rejections of the hypothesis that all the return series follow Lo's short-term dependence null. BHPV denotes the Bonferroni–Hochberg upper bound on the p-value of this hypothesis for the asymptotic tests.

On the other hand, cross-sectional dependence in asset returns could lead one to suspect that the MRS test statistics are not independent across the panel of return series. To address this issue we employ Bonferroni–Hochberg upper-bound p-values (Hochberg, 1988) of the hypothesis that all of the return series conform to Lo's short-term dependence null. These upper bounds hold regardless of whether or not the test statistics are dependent. Letting s^* denote the Bonferroni–Hochberg upper-bound p-value, this upper bound is given by

$$s^* = \min_{1 \leq j \leq N} \left[(N-j+1) P_j \right], \tag{12}$$

where $\{P_j\}$ denotes the p-values of test statistics ordered from lowest to highest. Table 3 shows these p-values corresponding to the asymptotic tests. [12] Only for the left-tailed tests based on $q^*(n)$ are the p-values less than conventional significance levels.

[12] These p-values are not computed for the bootstrapped tests. Given that the bootstrap tests are bases on 1,000 time-scrambled shufflings of the return series, the corresponding p-values are bounded by $P_j < 0.001$ for all $j = 1,\ldots,N = 1952$. As a result, the Bonferroni–Hochberg p-values have essentially no power in rejecting the null when bases on the bootstrapped critical values.

Fig. 2. Bootstrapped 95% critical values of the MRs test statistic $Q(n, q)$.

(A). For q-lengths $n^{1/4}$, $n^{1/3}$, and $n^{1/2}$. The curves along the (n, q) plane correspond to the 95% asymptotic critical value of 1.747. Looking from left to right, the plots correspond to $q = n^{1/4}$, $n^{1/3}$, and $n^{1/2}$.

(B). For q-lenghts of 90, 180, 270, and 360. The lines along the (n, q) plane correspond to the 95% asymptotic critical value of 1.747.

Because all the rejection rates are relatively small and because the majority of the 5% rejection rates are not significantly different from 0.05 (at 1% significance), one conclusion which can be safely drawn from the results is that long-range persistent and antipersistent behavior is not a widespread characteristic of common stocks. [13] Moreover, given some of the results in Tables 1 and 2 and Fig. 1.A, it is unclear that the series for which the test statistics yield rejections of Lo's composite null do in fact display evidence of long memory. For this reason, we take a closer look into the nature of the MRS rejections in the next section.

4. A logit study of MRS test rejections

To provide further insight into the possible economic and statistical explanations for the rejections by the MRS test, we present in this section the results of an exploratory logit study of the event of a test rejection. The logit study is structured so that we can better assess whether and how the test is linked to characteristics of firms such as their return distributions, ex-post survival, size, systematic risk, and industry affiliation. This section begins with a description of the dependent and independent variables used in the logit study.

4.1. Dependent and independent variables

We restrict our attention in the logit study to test rejections relating to 5% right- and left-tailed critical values. For a given tail ($T = \{\text{Left Tail, Right Tail}\}$), q-length formula ($QLF = q_i(n_i)$), and asymptotic or bootstrapped critical-value type ($CVT = \{\text{Asymptotic, Bootstrapped}\}$), the dependent variable in the logit models, $REJECT_i(T, QLF, CVT)$, assumes a value of 1 (0, otherwise) if the MRS test statistic corresponding to (T, QLF, CVT) for stock i indicates a 5%-significant rejection. The averages across the sample of stocks of these *REJECT* variables are reported in the rejections rates of Table 3.

Several independent variables are used in the logit specifications to account for such rejections. The sample averages, standard deviations and cross-correlations of these independent variables are reported in Table 4. The independent variables include a constant term and both merger and failed-firm indicator variables, MRG_i and $FAIL_i$. The merger and failed-firm variables are respectively assigned a value of 1 (0, otherwise) if firm i is merged (delisting codes 200 and 300 in the 1991 Daily CRSP Files) and if the firm is either liquidated or dropped from its exchange

[13] This conclusion is confirmed by testing for long memory using the Geweke and Porter-Hudak (1983) test. Although not reported here, results based on the GPH test indicate that the proportion of return series which at 5% significance display evidence of long memory is roughly 5%. Moreover, at conventional significance levels, tests based on the GPH test cannot reject the hypothesis that all of the return series are short-term dependent.

Table 4
Sample averages, standard deviations, and cross-correlations between the variables used in the logit regressions[a]

Sample averages and standard deviations

	AVG	SSD
MRG	0.3150	0.4646
FAIL	0.0240	0.1533
L_ACT/7420	0.2783	0.3561
L_MRG/7420	0.1185	0.5545
L_FAIL/7420	0.0059	0.0457
SIZE·1000	0.6471	1.8400
BETA	0.8840	0.4229
ALPHA·1000	0.1995	0.8482
NTF	0.0234	0.0601
MFG	0.4344	0.4958
CT&U	0.0983	0.2978
R&WTR	0.0906	0.2872
FI&RE	0.2223	0.4159
SRV	0.0686	0.2529

Cross-correlations

	MFG	*CT&U*	*R&WTR*	*FI&RE*	*SRV*
MRG	0.20*	−0.03	0.04	−0.21*	−0.01
FAIL	−0.08*	−0.03	−0.01	0.09*	0.02
L_ACT	0.02	0.19*	−0.04	−0.05*	−0.07*
L_MRG	0.18*	0.00	0.03	−0.18*	−0.04
L_FAIL	−0.06*	−0.03	−0.02	0.09*	0.01
SIZE	0.07*	0.04*	−0.03	−0.06*	−0.05*
BETA	0.24*	−0.13*	0.05*	−0.24*	0.08*
ALPHA	0.05*	0.02	0.00	−0.09*	0.04
NTF	0.01	−0.09*	0.02	0.07*	0.00
MXM	−0.04	0.04	−0.03	0.08*	−0.06*
MFG		−0.29†	−0.28†	−0.47†	−0.24†
CT&U			−0.10†	−0.18†	−0.09†
R&WTR				−0.17†	−0.09†
FI&RE					−0.15†

	FAIL	*L_ACT*	*L_MRG*	*L_FAIL*	*SIZE*	*BETA*	*ALPHA*	*NTF*	*MXM*
MRG	−0.11†	−0.53†	0.78†	−0.09†	−0.06*	0.20*	0.12*	−0.07*	−0.06
FAIL		−0.12†	−0.08†	0.83†	−0.04	−0.03	0.11	0.02	−0.13*
L_ACT			−0.41†	−0.10†	0.32*	−0.04	−0.03	−0.15*	0.11*
L_MRG				−0.07†	0.00	0.13*	0.07*	−0.10	−0.02
L_FAIL					−0.03	−0.02	0.07*	0.00	−0.10*
SIZE						0.09*	−0.02	−0.12*	0.11*
BETA							−0.02	−0.31*	0.01
ALPHA								0.01	−0.03
NTF									−0.19*

[a] Asterisks denote 5% nominally significant cross-correlations. Daggers denote correlations between variables which are dependent by construction.

(delisting codes 400–700). Firm-lifetime variables for active, merged, and failed firms (L_ACT_i, L_MRG_i, and L_FAIL_i) are also used. [14] L_MRG_i and L_FAIL_i assume values of $n_i \cdot MRG_i$ and $n_i \cdot FAIL_i$. Similarly, L_ACT_i is defined as $n_i \cdot ACT_i$ where ACT_i is an indicator variable which takes on a value of 1 (0, otherwise) if the firm is listed as active on its exchange at the end of 1991 (delisting code 100).

Industry-affiliation dummy variables based on one-digit SIC codes are also included in the logit models. The variables MFG_i, $CT\&U_i$, $R\&WTR_i$, $FI\&RE_i$, and SRV_i take on a value of 1 (0 otherwise) if firm i is affiliated with the manufacturing industry (one-digit SIC codes 2 and 3), the communications, transportation, and utilities industries (code 4), retail and wholesale trade (code 5), finance, insurance, and real estate (code 6), and services (codes 7 and 8).

The variable $SIZE_i$ denotes the average (over the lifetime of the ith firm) of the ratio of the market value of firm i's shares outstanding to the sum of the market value of the shares outstanding for all the firms in the sample which traded during the ith firm's lifetime. To describe the $SIZE_i$ variable more precisely, let $I_i(t)$ denote an indicator function which assigns a value of 1 (0, otherwise) if $t \in T_i$, where T_i is defined in Section 3.1. Also, let the number of firm i's shares outstanding at time t be denoted by $s_{i,t}$. The $SIZE_i$ variable is then given by

$$SIZE_i = [n_i]^{-1} \sum_{t \in T_i} (p_{i,t} \cdot s_{i,t}) \left(\sum_{j=1}^{N} I_j(t) \cdot p_{j,t} \cdot s_{j,t} \right)^{-1}. \tag{13}$$

The variables $ALPHA_i$ and $BETA_i$ respectively denote a risk-adjusted average excess (or net-risk-free) return on stock i and a variable reflecting firm i's exposure to systematic risk. $BETA_i$ is defined as the ratio of the sample covariance of firm i's excess returns with the CRSP value-weighted market excess returns to that of the sample variance of the market excess returns. The daily returns on three-month Treasury bills serve as a proxy for the risk-free rate. For clarity, let $\{r'_{i,t}\}$ and $\{r'_{M,t}\}$ denote the excess return series of stock i and the market proxy. $BETA_i$ can then be expressed as

$$BETA_i = \left(\sum_{t \in T_i} [r'_{i,t} - \mu_i(n_i)] \cdot (r'_{M,t} - \mu_M(n_i))] \right)$$
$$\times \left(\sum_{t \in T_i} [r'_{M,t} - \mu_M(n_i)]^2 \right)^{-1}, \tag{14}$$

[14] Recalling the notation in Section 3.1, for $\{t = 1, 7420\} \notin T_i$, the firm-lifetime variables yield an accurate value for the lifetimes of a merged or failed firm. For $\{t = 1\} \notin T_i$ and $\{t = 7420\} \notin T_i$, these variables yield an accurate value for the length of existence of an active firm up to December 31, 1991. Alternatively for $\{t = 1\} \in T_i$, the variables give a downward biased value of the lifetime of the firm.

where $\mu_i(n_i)$ and $\mu_M(n_i)$ denote the sample means of the return series corresponding to firm i and the market over $t \in T_i$. The average risk-adjusted excess return, $ALPHA_i$, is then given by

$$ALPHA_i = \mu_i(n_i) - B_i \cdot \mu_M(n_i). \tag{15}$$

Clearly, these *ALPHA* and *BETA* variables are very crude estimators of risk-adjusted average returns and market-risk exposures. Our interpretation of results based on these variables bear this point in mind.

Lastly, two other independent variables are included in the logit model, viz. NTF_i, and MXM_i. NTF_i denotes the proportion of trading days during which firm i's stock was not traded, and MXM_i denotes the 5% right-tailed Loretan and Phillips (1994) maximal-moment estimate associated with firm i's return series. [15]

4.2. Results

Table 5 reports the maximum likelihood parameter estimates and their asymptotic t-statistics for the logit models. [16] Although logit models for the rejections corresponding to each of the sample-size dependent q-lengths were estimated, we choose to report the results only for the models corresponding to the q-lengths of $n_i^{1/4}$ and $q^*(n_i)$. The results for the other cases are qualitatively similar. The conclusions we draw from the statistical results of the logit study are based on a nominal two-sided significance level of 5%.

The results in Table 5 indicate that the merger, lifetime of merged firms, risk-adjusted average return, and maximal-moment variables significantly influence the event of a left-tailed rejection by the MRS test across the *QLF* and *CVT* cases considered. In addition, for the *QLF* cases of $q_i = n_i^{1/4}$ the results indicate that the market-risk variable, *BETA*, also influences the event of a left-tailed rejection. The effects on the probability of a left-tailed rejection are positive for the *MRG* and *MXM* variables, and are negative for the others. The results across the *QLF* and *CVT* cases also indicate that the probability of a right-tailed rejection is significantly influenced by the maximal-moment and risk-adjusted average return variables. The effect on the probability of a right-tailed rejection is negative for *MXM* and is positive for *ALPHA*.

[15] See Loretan and Phillips, 1994 (Section 3). Although not shown here, the proportions of firms for which the hypothesis $\beta \geq 4$ is rejected at 5% significance are respectively 0.784 and 0.934 corresponding to the left- and right-tailed Loretan–Phillips maximal-moment estimates. On the bases of these results, fourth moments do not appear to exist for majority of common-stock returns.

[16] We use the BFGS algorithm (see Press et al., 1988, pp. 324–324) to estimate the parameters and the asymptotic standard errors of the parameters estimates.

Table 5
Maximum likelihood parameter estimates and asymptotic t-statistics for logit models of the MRS test rejections at the 5% significance level using asymptotic and bootstrapped critical values[a]

Variable	q: $q^*(n)$				q: $n^{1/4}$			
	Asymptotic		Bootstrapped		Asymptotic		Bootstrapped	
	Estimate	T value	Estimate	T value	Estimate	T value	Estimate	T value
5% left-tail rejections								
Intercept	−3.102	−5.65*	−3.578	−6.12*	−3.75	−7.86*	−3.853	−6.26*
MRG	1.262	2.97*	1.060	2.29*	1.471	3.47*	1.487	2.77*
FAIL	30.312	0.94	32.457	1.05	26.457	1.41	28.652	0.98
L_ACT/7420	−0.083	−0.23	−0.061	−0.017	0.288	1.25	0.561	1.32
L_MRG/7420	−5.638	-3.50*	−5.098	−3.13*	−5.378	−3.86*	−5.255	−2.73*
L_FAIL/7420	−276.930	−0.91	−293.223	−1.00	−239.150	−1.38	−250.580	−0.91
SIZE·1000	0.024	0.59	0.005	0.11	0.008	0.09	−0.013	−0.23
BETA	−0.397	−1.52	−0.376	−1.34	−0.672	−1.96*	−0.743	−2.26*
ALPHA·1000	−0.368	−2.43*	−0.318	−2.12*	−0.473	−2.99*	−0.257	−1.69
NTF	−0.087	−0.05	−2.456	−1.04	0.003	0.00	−0.876	−0.39
MXM	0.321	2.62*	0.381	3.08*	0.466	5.01*	0.497	3.97*
MFG	0.161	0.45	0.303	0.76	0.337	1.22	0..018	0.04
CT&U	0.076	0.17	0.207	0.42	−0.378	−0.57	−0.700	−1.25
R&WTR	−0.649	−1.25	−0.723	−1.20	−0.294	−0.47	−1.109	−1.64
FI&RE	−0.592	−1.49	−0.506	−1.15	−0.509	−1.28	−0.683	−1.50
SRV	0.434	0.97	0.702	1.47	0.838	3.34*	0.247	0.44
5% right-tail rejections								
Intercept	−1.604	−1.94	−1.125	−1.57	−1.556	−1.89	−1.042	−1.34
MRG	−0.379	−0.86	−0.213	−0.52	−0.607	−1.34	−0.304	−0.73
FAIL	−0.909	−0.81	−1.045	−1.02	−0.729	−0.68	−1.155	−1.11
L_ACT/7420	−0.177	−0.42	−0.269	−0.73	−0.087	−0.21	−0.094	−0.26
L_MRG/7420	−0.821	−0.84	−0.700	−0.76	0.062	0.06	−0.393	−0.42
L_FAIL/7420	3.835	1.35	3.809	1.45	3.540	1.26	4.036	1.52
SIZE·1000	−0.011	−0.15	−0.050	−0.50	−0.022	−0.24	−0.054	−0.55
BETA	0.092	0.33	0.011	0.04	0.328	1.17	0.279	1.15
ALPHA·1000	0.565	4.56*	0.615	5.08*	0.507	3.75*	0.658	5.56*
NTF	2.163	1.49	1.237	1.13	−0.130	−0.10	1.885	1.33
MXM	−0.568	2.09*	−0.589	−2.62*	−0.600	−2.14*	−0.700	−2.77*
MFG	−0.337	−0.79	−0.384	−0.98	−0.483	−1.15	−0.498	−1.41
CT&U	0.796	1.68	0.582	1.34	0.579	1.24	0.624	1.55
R&WTR	0.325	0.68	0.066	0.14	−0.137	−0.27	0.028	0.06
FI&RE	−0.307	−0.70	−0.139	−0.34	−0.427	−0.98	−0.239	−0.63
SRV	0.000	0.00	−0.077	−0.15	−0.127	−0.24	−0.215	−0.45

[a] The dependent variables correspond to tests based on q-lengths of $q = n^{1/4}$ and $q = q^*(n)$. Asterisks denote 5% nominally significant parameter values corresponding to a two-sided test.

4.3. Discussion

If the event of a left-tailed MRS test rejection were the result of survivorship bias, we would expect positive signs on the coefficients attached to the *MRG* and

FAIL variables and negative signs on the lifetime variables *L_ACT*, *L_MRG*, and *L_FAIL* in the logit models of the left-tailed rejections. With a few exceptions this is indeed the case, but only for the merger variables are the coefficients significant. We conclude, then, that there is some evidence linking the left-tailed rejections to survivorship bias.

We are unable to provide a satisfactory explanation for the significantly negative coefficients on the *BETA* variable in some of the logit models of the left-tailed rejections. To the extent that the *BETA* variable measures the exposures of the firms to market risk and perhaps therefore to their prospects for survival, we would expect positive signs on the *BETA* coefficients. [17] Instead, we simply conclude that the event of a left-tailed rejection by the MRS test appears to be linked to firms with relatively low market risk.

If the sole effect of the *MXM* variable were to indicate FMC failures, we would expect insignificant coefficients on *MXM* in the logit models of the bootstrapped test rejections, and negative (positive) signs on the coefficients of *MXM* in the logit models of the left-tailed (right-tailed) asymptotic test rejections (cf. Section 2.5). Instead, the coefficients on the *MXM* variable in the logit models of the left-tailed (right-tailed) asymptotic and bootstrapped MRS test rejections are all significantly positive (negative). As a result, FMC failures cannot account for the net effect of the *MXM* variable on the tests. [18] For this reason, we therefore conclude that the *MXM* variable may proxy for some other firm characteristic which manifests itself in the MRS tests by shifting the test distributions *to the right* of their corresponding null distributions. Whether or not this effect is an indicator of persistent long memory is unclear. We leave for further study a closer examination of this question.

And lastly, the return series of firms with relatively high risk-adjusted average returns are more (less) likely to be rejected by right-tailed (left-tailed) tests. To the extent that large *ALPHA* values represent market mispricing, and if the right-tailed rejections indicate persistent long memory, this result suggests a possible link between right-tailed MRS test rejections, persistent long memory, and market mispricings in a small fraction common stocks. But, on the basis of our analysis, we emphasize that such a link is highly speculative.

[17] See, for example, Chan and Chen (1991) and Queen and Roll (1987) for studies were firm performance is linked to other firm characteristics. The results of Chan and Chen suggest that a firm's market-risk exposure should be positively linked to poor performance, while Queen and Roll conclude that market risk is a poor forecaster of firm mortality.

[18] This result is confirmed in an alternative way. There are 65 firms in our panel for which the Loretan–Phillips tests of the hypothesis $\beta \leq 4$ cannot be rejected at 5% significance. Although not reported here, there are no excessive right-tailed MRS test rejection rates corresponding to the return series of these firms. Alternatively, the majority of rejection rates associated with the bootstrapped and asymptotic MRS tests based on q-lengths of $q^*(n)$ and $n^{1/4}$ are excessive at 5% significance.

When taken as a whole the excessive bootstrapped right-tailed rejection rates, the connection of the right-tailed rejections to large risk-adjusted average returns, and the apparent freedom of the right-tailed rejections from survivorship and MCF effects, we conclude that there is some evidence to suggest that the relatively few return series which yield right-tailed rejections appear tied to persistent long memory. [19]

5. Summary

Fama (1991) argues that research on market efficiency should be evaluated in terms of whether it improves our ability to describe the time-series and cross-sectional variation of security returns. Much recent work has addressed the issue of the presence of slowly-decaying components in stock prices because of the controversial implications of such a finding for martingale models of asset prices used in financial economics. This work has produced mixed results. The frequently-cited studies by Fama and French (1988) and Poterba and Summers (1988) find evidence of long horizon mean-reverting behavior in stock returns which is consistent with fads, irrationality, speculative bubbles, and noise trading. Brown et al. (1993), Kim et al. (1991), Richardson (1993), and Richardson and Stock (1989) among others suggest, however, that these results showing long-run behavior may be spurious. Moreover, Kandel and Stambaugh (1989) argue that more conventional models of short-term dependence in stock returns can explain the long-run findings of Fama and French and Poterba and Summers. And, the time-series models which underpin their findings are consistent with the Lo (1991) short-term dependence null.

The contribution of this paper is to provide additional insight into this debate by focusing on the time-series behavior of daily returns on common stocks. Previous studies have for the most part examined aggregate stock returns, with the exception of Greene and Fielitz (1977). Moreover, we focus on a particular type of long-term dependence in asset returns, viz. long memory, which appears unreconcilable with market efficiency. We apply Lo's MRS test for long memory to the daily returns of 1,952 common stocks. In contrast to the conclusions of Greene and Fielitz in particular and those of Peters (1989, 1991, 1992, 1994) in general, we find that long memory is not a widespread characteristic of common stocks.

We also find that the MRS test is sensitive to the choice of autocovariance lag-truncation length, to moment-condition failures, and to survivorship bias.

[19] An alternative interpretation of the excessive right-tailed rejection rates associated with the bootstrapped tests is that they possibly stem from shifts in the means of the returns series. As previously mentioned, both the GPH and MRS tests are very sensitive to shifts in means. We therefore consider this alternative view unlikely, since the rejection rates corresponding to the right-tailed asymptotic MRS tests and GPH tests do not indicate excessive rejections.

Moreover, estimates from logit models of the event of a rejection by the MRS test generate some results which are consistent with the effects of survivorship bias in the MRS test statistics associated with our panel of return series.

Alternatively, the results from our logit study also indicate that the returns of firms with heavy-tailed distributions and those with large risk-adjusted average returns are more likely to generate right-tailed MRS test rejections. We conclude that the relatively few return series which are rejected by the right-tailed test appear to be tied to persistent long memory.

Acknowledgements

We wish to thank Dan Asquith, William A. Brock, Pedro de Lima, Paul Draper, Larry Harris, Joseph Haubrich, Chris Jones, Charles Kramer, Douglas Lamdin, Blake LeBaron, Andrew Lo, Francis Longstaff, Robert McClelland, James Poterba, Simon Potter, N.R. Prabhala, Gerard Pfann (the editor), and two anonymous referees for their helpful comments on this study. The authors are solely responsible, however, for any remaining errors. Drafts of this paper were presented at the Santa Fe Institute, the University of Stirling, the Symposium on Nonlinear Dynamics at the University of Strathclyde, the Money, Macro, and Finance Research Group held at the London Business School, and at the 1995 American and 1996 European Finance Association Conferences. The authors also wish to note that the Securities and Exchange Commission, as a matter of policy, disclaims responsibility for any private publication or statement of its employees. The views expressed here are those of the authors and do not necessarily represent the views of the Commission.

References

Agiakloglou, C. and P. Newbold, 1994, Lagrange multiplier tests for fractional difference, Journal of Time Series Analysis 15, 253–262.

Ambrose, B., E. Ancel and M. Griffiths, 1993, Fractal structure in the capital markets revisited, Financial Analysts Journal, May/June, 73–77.

Andrews, D., 1991, Heteroskedasticity and autocorrelation consistent covariance matrix estimation, Econometrica 59, 817–858.

Baillie, R., 1995, Long memory processes and fractional integration in econometrics, Journal of Econometrics, forthcoming.

Baillie, R., T. Bollerslev and H. Mikkelsen, 1993, Fractionally integrated generalized autoregressive conditional heteroskedasticity, Working paper no. 168 (Department of Finance, Northwestern University, Evanston, IL).

Bollerslev, T. and H. Mikkelsen, 1993, Modeling and pricing long-memory in stock market volatility, Working paper no. 134 (Department of Finance, Northwestern University, Evanston, IL).

Booth, G. and F. Kaen, 1979, Gold and silver spot prices and market information efficiency, Financial Review 14, 21–26.

Booth, G., F. Kaen and P. Koveos, 1982a, Persistent dependence in gold prices, Journal of Financial Research 5, 85–93.

Booth, G., F. Kaen and P. Koveos, 1982b, R/S analysis of foreign exchange rates under two international monetary regimes, Journal of Monetary Economics 10, 407–415.

Breidt, J., N. Crato and P. de Lima, 1994, Modeling long memory stochastic volatility, Working papers in economics no. 323 (Department of Economics, Johns Hopkins University, Baltimore, MD).

Brock, W. and P. de Lima, 1995, Nonlinear time series, complexity theory, and finance, In: G.S. Maddala and C.R. Rao, eds., Handbook of statistics, Vol. 14 (North Holland, Amsterdam).

Brown, S., W. Goetzmann and S. Ross, 1993, Survivorship bias in autocorrelation and long-term memory studies, Unpublished manuscript (Graduate School of Business, Columbia University, New York).

CRSP (Center for Research in Securities Prices), 1992, CRSP stock file guide (Graduate School of Business, University of Chicago, Chicago, IL).

Chan, K. and N. Chen, 1991, Structural and return characteristics of small and large firms, Journal of Finance 46, 1467–1484.

Cheung, Y., 1993, Tests for fractional integration: A Monte Carlo investigation, Journal of Time Series Analysis 14, 331–345.

Cheung, Y. and K. Lai, 1993, Do gold markets have long memory?, Financial Review 28, 181–202.

Cheung, Y., K. Lai, and M. Lai, 1993, Are there long cycles in foreign stock returns?, Journal of International Financial Markets, Institutions and Money 3, 33–47.

Crato, N., 1994, Some international evidence regarding the stochastic memory of stock returns, Applied Financial Economics 4, 33–39.

Crato, N. and P. de Lima, 1994, Long range dependence in the conditional variance of stock returns, Economics Letters 45, 281–285.

Davies, R. and D. Harte, 1987, Tests for Hurst effect, Biometrika 74, 95–102.

Ding, Z., C. Granger and R. Engle, 1993, A long memory property of stock market returns and a new model, Journal of Empirical Finance 1, 83–106.

Fama, E., 1991, Efficient capital markets: II, Journal of Finance 46, 1575–1617.

Fama, E. and K. French, 1988, Permanent and temporary components of stock prices, Journal of Political Economy 96, 246–273.

Geweke, J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, Journal of Time Series Analysis 4, 221–238.

Goetzmann, W., 1993, Patterns in three centuries of stock market prices, Journal of Business 66, 249–269.

Granger, C., 1980, Long memory relationships and the aggregation of dynamic models, Journal of Econometrics 14, 227–238.

Granger, C. and R. Joyeux, 1980, An introduction to long-memory time series models and fractional differencing, Journal of Time Series Analysis 1, 15–29.

Greene, M. and B. Fielitz, 1977, Long-term dependence in common stock returns, Journal of Financial Economics 4, 339–349.

Harvey, A., 1993, Long memory in stochastic volatility, Unpublished manuscript (London School of Economics, London).

Helms, B., F. Kaen and R. Rosenman, 1984, Memory in commodity futures contracts, Journal of Futures Markets 4, 559–567.

Hochberg, Y., 1988, A sharper Bonferroni procedure for multiple tests of significance, Biometrika 75, 800–902.

Hodges, S., 1995, Arbitrage in a fractal Brownian motion market, Unpublished manuscript (Options Research Centre, University of Warwick, Coventry).

Hosking, J., 1981, Fractional differencing, Biometrika 68, 165–176.

Hurst, H., 1951, Long term storage capacity of reservoirs, Transactions of the American Society of Civil Engineers 116, 770–779.

Kandel, S. and R. Stambaugh, 1989, Modelling expected stock returns for long and short horizons, Working paper no. 42-88 (Rodney L. White Center, Wharton School, University of Pennsylvania, Philadelphia, PA).

Kim, M., C. Nelson and R. Startz, 1991, Mean reversion in stock prices? A reappraisal of the empirical evidence, Review of Economic Studies 58, 515–528.

Lo, A., 1991, Long-term memory in stock market prices, Econometrica 59, 1279–1313.

Loretan, M. and P. Phillips, 1994, Testing the covariance stationarity of heavy-tailed time series: An overview of the theory with applications to several financial data sets, Journal of Empirical Finance 1, 211–248.

Mandelbrot, B., 1971, When can price be arbitraged efficiently? A limit to the validity of the random walk and martingale models, Review of Economics and Statistics 53, 225–236.

Mandelbrot, B. and J. Van Ness, 1968, Fractional Brownian motion, fractional noises and applications, S.I.A.M. Review 10, 422–437.

Mandelbrot, B. and J. Wallis, 1969, Robustness of rescaled range R/S in the measurement of noncyclic long run statistical dependence, Water Resources Research 5, 967–988.

Mills, T., 1993, Is there long-term memory in UK stock returns?, Applied Financial Economics 3, 303–306.

Peters, E., 1989, Fractal structure in the capital markets, Financial Analysts Journal, July/Aug., 32–37.

Peters, E., 1991, Chaos and order in the capital markets (Wiley, New York).

Peters, E., 1992, R/S analysis using logarithmic returns, Financial Analysts Journal, Nov./Dec., 81–82.

Peters, E., 1994, Fractal market analysis (Wiley, New York).

Phillips, P., 1987, Time series regression with a unit root, Econometrica 55, 277–301.

Poterba, J. and L. Summers, 1988, Mean reversion in stock prices, Journal of Financial Economics 22, 27–60.

Press, W., S. Tuekolsky, W. Vetterling and B. Flannery, 1988, Numerical recipes in C: The art of scientific computing (Cambridge University Press, Cambridge).

Queen, M. and R. Roll, 1987, Firm mortality: Using market indicators to predict survival, Financial Analysts Journal, May/June, 9–26.

Richardson, M., 1993, Temporary components in stock prices: A skeptic's view, Journal of Business and Economic Statistics 11, 199–207.

Richardson, M. and J. Stock, 1989, Drawing inferences from statistics based on multiyear asset returns, Journal of Financial Economics 25, 323–348.

Robinson, P., 1991, Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression, Journal of Econometrics 47, 67–84.

Wallis, J. and N. Matalas, 1970, Small sample properties of H and K, estimators of the Hurst coefficient h, Water Resources Research 6, 1583–1594.