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Real and Spurious Long-Memory Properties of Stock-Market Data

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We test for the presence of long memory in daily stock returns and their squares using a robust semiparametric procedure of Lobato and Robinson. Spurious results can be produced by nonstationarity and aggregation. We address these problems by analyzing subperiods of returns and using individual stocks. The test results show no evidence of long memory in the returns. By contrast, there is strong evidence in the squared returns.

KEY WORDS: Lagrange multiplier test; Long-range dependence; Semiparametric procedure.

There have been several works analyzing the long-term properties of stock returns. Greene and Fielitz (1977) used the R/S statistic (Hurst 1951) to test for long-term dependence in the daily returns of 200 individual stocks on the New York Stock Exchange from December 23, 1963, to November 29, 1968, and claimed to have found significant evidence. Lo (1991) criticized these results on the grounds that this evidence was due to short-term correlation. He proposed a modified version of the R/S statistic to test robustly for long-term dependence and found no evidence in favor of long-run dependence of the monthly and daily returns on Center for Research in Security Prices (CRSP) stock indexes. Ding, Granger, and Engle (1993) examined the long-memory properties of several transformations of the absolute value of daily returns on the Standard and Poor's (S&P) 500, including squared returns, and found considerable evidence of long memory in the squared returns but conducted no formal test.

The purpose of this article is twofold. The first is to conduct a formal test using a semiparametric procedure due to Lobato and Robinson (1997). They applied the test to exchange-rate data, including squares of changes in exchange rates. The null hypothesis is that of weak dependence or short memory, the alternative being strong dependence or long memory. The procedure focuses on the long-memory properties of the data irrespective of the short-term dependence. Although the R/S procedures are robust, their efficiency properties are questionable; see Robinson (1994). Furthermore, the test statistic used by Lo has a complicated asymptotic distribution when the null is true, whereas the test statistic we consider has the convenient feature that its asymptotic distribution is chi-squared. Our test accepts weak dependence for daily returns on the S&P 500 but rejects for squared returns. The rejection is even stronger for absolute returns.

The second purpose is to investigate whether rejection of weak dependence is due to long memory or is due to other causes. Two common causes of spurious long memory are nonstationarity and aggregation. Nonstationarity is a plausible explanation for our findings, especially those of Ding et al., who used S&P 500 data from 1928 to 1992. During this period, there were changes in the mean of squared returns. It was very high in the early thirties and then was much reduced by the end of the decade. During the mid-seventies and the eighties, there was a substantial increase in the mean of the squared returns, perhaps due to factors such as the introduction of new financial products and the widespread use of computer trading programs; see, for example, Grossman and Zhou (1996). The mean of squared returns appears to have decreased again in the nineties. Changes in the mean of squared returns also occur for individual stocks.

In the case of stock indexes, the evidence in favor of long memory may be due to the effect of aggregation. The key idea is that aggregation of independent weakly dependent series can produce a strong dependent series. For example, in the case of the squares of the daily returns of the S&P 500 it could happen that squares for the individual stocks do not exhibit long memory and the apparent long memory of the index is just due to aggregation. A motivation for this can be found, for instance, in the work of Robinson (1978) or Granger (1980).

We address the nonstationarity problem by splitting up the daily data into arguably stationary periods and the aggregation problem by using daily data on the individual stocks in the Dow Jones Industrial Average. Our conclusions confirm the results of Ding et al. (1993). In particular, for subseries of the S&P 500 index that appear stationary, our test favors long memory. Similar results are obtained for the subseries for the individual stocks in the Dow Jones Industrial Average.

The organization of the article is the following. In Section 1 we briefly review the concept of long memory and describe the procedure we use to test for long memory. Section 2 contains our analysis of the long-memory properties of the data. In Section 3 we discuss our results and comment on intradaily stock returns.

1. TEST STATISTIC

In this section we describe the test statistic that we employ to analyze the long-memory properties of the data.

There is no unique definition of a long-memory process. Consider a covariance stationary process x_t, assume that its spectral density function exists, and call it $f(\lambda)$. The condition

$$f(\lambda) \sim C\lambda^{1-2H} \quad \text{as } \lambda \to 0^+ \tag{1.1}$$

for $H < 1, H \neq 1/2$, with C a positive constant, characterizes x_t as a long-memory process. Notice that (1.1) includes two different cases. For $H \in (1/2, 1), f(\lambda)$ tends to infinity as it is evaluated at frequencies that tend to 0 (this is called the strictly long-memory case), but when $H < 1/2$, it tends to 0 (this is called the antipersistent case). The case $H = 1/2$ represents the weakly dependent case; $f(\lambda)$ tends to a constant as it is evaluated at frequencies that tend to 0.

In the time domain, long memory can be characterized as follows. Let γ_j denote the autocovariance at lag j of $x_t, \gamma_j = E[(x_1 - \mu)(x_{1+j} - \mu)]$, with μ denoting the mean of the process x_t. The condition

$$\gamma_j \sim Kj^{2H-2} \quad \text{as } j \to \infty, \tag{1.2}$$

where K is a constant and H takes the same values as previously, characterizes x_t as a long-memory process.

Conditions (1.1) and (1.2) are not necessarily equivalent, but for fractional autoregressive integrated moving average processes, both hold. Notice that when $H \in (1/2, 1)$ both conditions (1.1) and (1.2) imply

$$\sum_{j=-\infty}^{\infty} |\gamma_j| = \infty. \tag{1.3}$$

This condition is a more general definition of strictly long memory.

H is the parameter that determines the degree of long memory (the higher the H the longer the memory), so testing the null hypothesis of weak dependence against the alternative of long memory is equivalent to testing $H = 1/2$ against $H \neq 1/2$.

Notice that (1.1) only characterizes the behavior of $f(\lambda)$ in the neighborhood of 0 and nothing is specified about the medium- or short-term behavior of the process. Therefore, robust estimation and testing procedures in the frequency domain can be carried out using the periodogram (or some functions of the periodogram) evaluated in a degenerating neighborhood of zero frequency. To do so it is necessary to introduce a bandwidth number m that tends to infinity as the sample size (n) tends to infinity, but slowly so that m/n tends to 0.

Robinson (1995) analyzed a robust estimation procedure based on choosing C and H so as to minimize the following objective function (see Künsch 1987; Lobato and Robinson 1997):

$$Q(C, H) = \frac{1}{m} \sum_{j=1}^{m} \left(\log C\lambda_j^{1-2H} + \frac{\lambda_j^{2H-1}}{C} I(\lambda_j) \right), \tag{1.4}$$

where $I(\lambda_j)$ is the periodogram at frequency λ_j $(= 2\pi j/n)$,

$$I(\lambda_j) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} x_t e^{i\lambda_j t} \right|^2.$$

Robinson analyzed the properties of the estimate that minimize (1.4) in a compact set $[\Delta_1, \Delta_2]$ with $0 < \Delta_1 < \Delta_2 < 1$. Denoting this estimate by $\hat{H}$, he proved that

$$\sqrt{m}(\hat{H} - H) \to_d N\left(0, \frac{1}{4}\right). \tag{1.5}$$

This estimate appears to be the most efficient semiparametric estimate developed so far.

We employ an approximation to the Lagrange multiplier (LM) test to test $H = 1/2$ against $H \neq 1/2$ (or $H > 1/2$)

Table 1. LM Test for S&P 500

	m							
Series	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
			A. July 1962 to December 1994					
Returns	.76	.11	.02	.15	.07	.14	.12	.24
Squared returns	2.50	3.68	5.06*	6.14*	7.60*	10.3*	13.5*	17.0*
Absolute value of returns	21.8*	34.6*	45.6*	57.3*	74.4*	96.5*	120*	150*
			B. July 1962 to December 1972					
Returns	.00	.21	.00	.00	—	—	—	—
Squared returns	4.61*	9.00*	14.9*	23.8*	—	—	—	—
Absolute value of returns	12.9*	22.3*	34.4*	51.0*	—	—	—	—
			C. January 1973 to December 1994					
Returns	.08	.17	.09	.00	.12	.06	—	—
Squared returns	1.00	1.49	2.71	4.52*	7.01*	8.67*	—	—
Absolute value of returns	8.52*	12.3*	21.6*	35.5*	53.4*	72.4*	—	—

NOTE: * indicates significant at the 5% level.

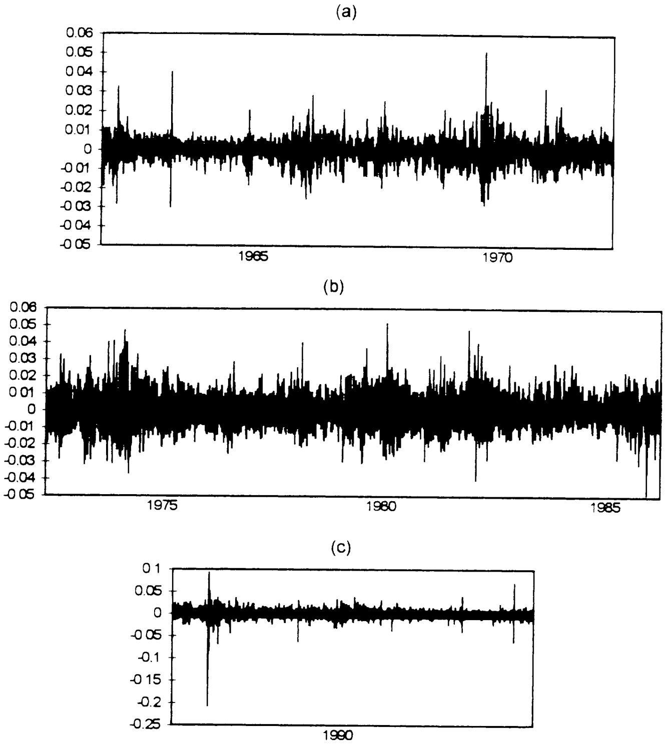

Figure 1. S&P 500 Daily Returns: (a) July 1962–December 1972; (b) January 1973–December 1986; (c) January 1987–December 1994.

based on the objective function (1.4). This test is a particular case of the more general test analyzed by Lobato and Robinson (1997). For the univariate case and a two-sided alternative hypothesis, the LM test statistic has the form

$$\mathrm{LM} = m\left(\frac{\sum_{j=1}^{m} v_j I(\lambda_j)^2}{\sum_{j=1}^{m} I(\lambda_j)}\right), \tag{1.6}$$

where

$$v_j = \log j - \frac{1}{m}\sum_{j=1}^{m} \log j.$$

The details of the objective function and the testing procedure were given by Lobato and Robinson (1997). They provided conditions that establish under the null hypothesis ($H = 1/2$) that LM $\rightarrow_d \chi_1^2$ and also conditions for the consistency of the test. Monte Carlo analysis of this test was also provided. An alternative procedure that sometimes produces slightly better finite-sample performance is to use the periodogram of tapered rather than observed data in Expression (1.6).

The Wald test can also be based on (1.4). The disadvantage of the Wald test is that it needs an estimate for H, so the minimization of (1.4) has to be carried out by iterative procedures. Monte Carlo analysis of this test was reported by Robinson (1995).

2. EMPIRICAL RESULTS

In Table 1 we report results for the LM test of long memory for daily returns, squared returns, and the absolute value of the returns for the S&P 500 index between July 1962 and December 1994. The sample size is n = 8,178. We report the test for a grid of values of m from $m = 30$ to $m = 100$. When m equals 30 and 100, the shortest periods that are taken into account by the test correspond to approximately 273 and 82 days, respectively. No evidence of long memory is found in the returns, but there is strong evidence of long memory in the squares. This evidence is even stronger for the absolute value of the returns, and hence we concentrate on the squares. These results are in agreement with Lo (1991) and Ding et al. (1993).

There are several ways in which the LM test for long memory can produce spurious results. First, the squared returns process could possess a shift in the mean. To show how this can happen, consider the following setup. Let $y_t, t = 1, 2, \ldots, N$, be a zero mean stochastic process with

$$Ey^2 = \begin{cases} \sigma_1^2 & t = 1, \ldots N_b \\ \sigma_2^2 & t = N_b + 1, \ldots, N, \end{cases}$$

where y_t is independent of y_s for $t \neq s$ and, for some $v > 0, Ey_t^{2+v} < \infty$ for all t. Denote the sample autocovariances for y or y^2 by

$$\hat{\gamma}_{a,j} = \frac{1}{N}\sum_{t=1}^{N-j} (a_t - \bar{a})(a_{t+j} - \bar{a}), \qquad \bar{a} = \frac{1}{N}\sum_{t=1}^{N} a_t$$

for $j = 1, \ldots, N-1$ and $a = y$ or y^2.

Obviously y_t and y_t^2 are not covariance stationary processes. Hence, the definitions of long memory stated in Section 1 do not apply. Nevertheless, imagine that the researcher does not know this and tests for evidence of long memory in y_t and y_t^2. What would he/she find? First con-

Table 2. List of Companies With Starting and Ending Dates

Tick	Name	Starting date	Ending date
ATT	AT&T Corp.	07–02–1962	12–30–1994
ALD	Allied Signal Inc.	07–02–1962	12–30–1994
AA	Aluminum Company Amer.	07–02–1962	12–30–1994
AXP	American Express Co.	05–18–1977	12–30–1994
BS	Bethlehem Steel Corp.	07–02–1962	12–30–1994
BA	Boeing Co.	07–02–1962	12–30–1994
CAT	Caterpillar Inc.	07–02–1962	12–30–1994
CHV	Chevron Corp.	07–02–1962	12–30–1994
KO	Coca Cola Co.	07–02–1962	12–30–1994
DIS	Walt Disney Co.	07–02–1962	12–30–1994
DD	Du Pont E. I. De Nemours & Co.	07–02–1962	12–30–1994
EK	Eastman Kodak Co.	07–02–1962	12–30–1994
XON	Exxon Corp.	07–02–1962	12–30–1994
GE	General Electric Co.	07–02–1962	12–30–1994
GM	General Motors Corp.	07–02–1962	12–30–1994
GT	Goodyear Tire & Rubber Co.	07–02–1962	12–30–1994
IBM	IBM	07–02–1962	12–30–1994
IP	International Paper Co.	07–02–1962	12–30–1994
MCD	McDonalds Corp.	07–05–1966	12–30–1994
MRK	Merck & Co. Inc.	07–02–1962	12–30–1994
MMM	Minnesota Mining & Mfg. Co.	07–02–1962	12–30–1994
JPM	J. P. Morgan & Co. Inc.	04–01–1969	12–30–1994
MO	Philip Morris Cos. Inc.	07–02–1962	12–30–1994
PG	Procter & Gamble Co.	07–02–1962	12–30–1994
S	Sears Roebuck & Co.	07–02–1962	12–30–1994
TX	Texaco Inc.	07–02–1962	12–30–1994
UK	Union Carbide Corp.	07–02–1962	12–30–1994
UTX	United Technology Corp.	07–02–1962	12–30–1994
WX	Westinghouse Electric Corp.	07–02–1962	12–30–1994
Z	Woolworth Corp.	07–02–1962	12–30–1994

Table 3. LM Test for Stock Returns for July 1962–December 1972

	m				m			
	30	40	50	60	30	40	50	60
Stock	Returns				Squared returns			
ATT	.18	.00	.00	.00	4.05*	3.11	2.55	1.38
ALD	.05	.62	.47	.21	30.6*	40.7*	46.9*	58.0*
AA	.12	.57	1.01	.31	23.4*	32.0*	38.3*	50.5*
AXP	—	—	—	—	—	—	—	—
BS	.54	1.32	1.24	.70	9.29*	11.2*	12.4*	10.3*
BA	2.54	4.64	5.31*	1.26	34.0*	55.5*	61.0*	64.7*
CAT	.00	.00	.17	.15	4.94*	7.99*	12.3*	13.4*
CHV	2.09	2.14	1.09	.01	9.60*	17.7*	17.6*	19.3*
KO	.08	.07	.10	.00	12.6*	13.6*	26.8*	31.5*
DIS	.54	.00	.01	.02	16.8*	23.6*	40.3*	45.5*
DD	2.85	6.19*	1.94	1.19	11.9*	19.5*	30.1*	36.8*
EK	.35	.43	.12	.05	18.7*	23.7*	36.0*	41.4*
XON	5.13*	3.61	1.11	.57	6.90*	12.4*	12.1*	16.2*
GE	.49	.09	.02	.13	4.90*	7.90*	12.8*	14.7*
GM	.00	.16	.07	.04	.67	1.37	.98	2.31
GT	4.06	3.07	2.80	1.77	25.7*	41.9*	42.9*	40.9*
IBM	.00	1.02	.92	1.11	6.32*	11.6*	17.9*	30.5*
IP	3.81	3.75	3.31	2.49	6.92*	4.41*	4.80*	5.78*
MCD	.19	.78	.56	.38	9.36*	13.2*	18.9*	25.3*
MRK	.01	.02	.04	.44	2.52	3.85*	2.31	2.38
MMM	.32	.05	.14	.03	3.98*	7.66*	11.7*	21.4*
JPM	.13	.00	.03	.64	5.23*	2.02	1.05	1.33
MO	.76	.25	.21	.00	20.5*	31.4*	39.6*	31.5*
PG	.07	.09	.14	.02	8.10*	11.7*	18.1*	22.9*
S	.38	.67	1.02	.76	.69	.60	.10	.81
TX	.65	2.14	.95	.01	46.0*	58.4*	71.1*	85.1*
UK	.17	.49	1.58	1.11	18.0*	23.6*	27.0*	25.4*
UTX	.52	.02	.74	.23	49.3*	43.9*	48.6*	66.5*
WX	.88	.12	.02	.03	.78	.00	.16	.46
Z	.00	1.05	.23	.45	5.57*	6.77*	9.82*	12.4*

NOTE: * indicates significant at 5% level. Notice that MCD starts 660705 and JPM 690401.

sider the behavior of the sample autocovariances for y_t. It is immediate to show that

$$\hat{\gamma}_{y,j} = O_p\left(\frac{1}{\sqrt{N}}\right)$$

for all j, exactly the same as we get with a white-noise process, so long memory should not be detected. What about the behavior of y_t^2? In the appendix it is shown that, for some constants c_j different from 0, $\hat{\gamma}_{y^2,j} \rightarrow_p c_j$ for all j. So it is not that the sample autocovariances tend slowly to 0. In fact, they do not even tend to 0.

Nonstationarity may be responsible for the findings of Ding et al. (1993). They analyzed the S&P 500 series from 1928 to 1992. During this period there are several reasons to suspect nonstationarity. The squared returns appear to be much larger in the thirties than in later periods. The functioning of the stock market may have been affected by World War II. Recently, during the mid-seventies and especially the eighties, financial markets have seen the introduction of new financial products and a widespread use of information technology in the trading process. These considerations may be relevant to understand the Ding et al. (1993) results. Our sample goes from July 1962 to December 1994, so it also covers a period in which the introduction of financial innovations raises questions about the stationarity assumption.

To investigate the possibility that the observed evidence of long memory is, in fact, due to nonstationarity, we split our sample into two periods. We take January 1973 as the break point because the oil-price shock occurred in that year. In Table 1 we present the results of the LM test for long memory for the two subsamples using several values for m. In Figure 1 we plot S&P 500 returns for periods July 1962–December 1972, January 1973–December 1986, and January 1987–December 1994. From this plot there is a very clear increase in volatility between the first two periods. The crash in October 1987 dominates the bottom part of Figure 1. Our eyeball test says that the series is stationary during the period 1962–1972 and it may be for 1973–1994. Thus, it is of interest that the squared returns exhibit strong evidence of long memory for the period 1962–1972 as well as for 1973–1994. Hidalgo and Robinson (1996) gave a test for structural change in the mean in the presence of long memory when the time series is Gaussian. To our knowledge, no formal test is available for non-Gaussian processes. [The fact that stock returns are non-Gaussian has been established in several works; see, for instance, Brock and de Lima (1996).]

The second reason why the evidence of long memory in the squared returns in the S&P 500 can be spurious is based on aggregation. The S&P 500 is an aggregate index of the stock market, so its squared returns are derived from the squared returns of the individual stocks. It may well happen that the specific stocks do not exhibit strong dependence and the apparent long memory of the index is just due to aggregation. A motivation of this can be found, for instance, in the work of Robinson (1978) or Granger (1980). These articles showed that, starting with individual independent first-order autoregressive [AR(1)] series with random autoregressive coefficients, the aggregate series can exhibit long memory for certain specifications of the distribution function from which these coefficients are drawn. This result can be generalized to other weak dependent processes, in particular autoregressive moving average processes. The key idea is that aggregation of independent weakly dependent series can produce a strongly dependent series. In our case, it seems very implausible to assume independence of the squared returns processes for the individual stocks. So the aggregation explanation of Robinson (1978) and Granger (1980) is not directly applicable. Nonetheless, what is clear is that aggregation may produce spurious evidence of long memory.

To examine this possibility, we analyzed the long-memory properties of the 30 stocks that compose the Dow Jones Industrial Average. These 30 stocks are listed in Table 2 with their ticks and periods covered. Except for three cases, the data are for July 1962 to December 1994. These data are taken from the CRSP files. In Tables 3 and 4 we report the LM statistics for the returns and the squared returns for the periods July 1962 to December 1972 and January 1973 to December 1994.

There are several features that should be noticed. First, there is no evidence of long memory in the returns for any period. Second, with respect to the squares the evidence is more varied. For the period July 1962 to December 1972 all stocks but six (ATT, GM, MRK, JPM, S, WX) show strong evidence. For the period January 1973 to December 1994 there is stronger evidence of long memory in the squared returns. It is worth mentioning that the period 1973–1994 has gone through substantial changes in both financial instruments and information-technology tools. Thus, it is plausible that the squared returns are nonstationary in this period. This could explain the widespread finding of long memory in the squared returns in this period.

The results for the LM test for the whole period are in Table 5. It is not surprising that for all the series there is evidence of long memory in the squared returns. For some stocks, in particular the six just noted, this evidence of long memory may be spurious and may be due to nonstationarity during the whole period.

Nonstationarity and aggregation are two important causes of spurious evidence of long memory, but they are not the only ones. In the rest of the section we mention three additional causes.

Table 4. LM Test for Stock Returns for January 1973–December 1994

	m				*m*			
	30	*50*	*60*	*80*	*30*	*50*	*60*	*80*
Stock	*Returns*				*Squared returns*			
ATT	.02	.47	.36	.28	13.4*	22.3*	29.4*	34.7*
ALD	.00	1.16	.79	.27	.09	.88	1.38	2.02
AA	2.17	2.10	1.18	2.21	4.38*	13.2*	16.3*	29.1*
AXP	—	—	—	—	—	—	—	—
BS	1.10	.29	.10	1.52	26.9*	36.8*	41.3*	60.9*
BA	.14	.22	.17	.00	15.1*	34.7*	52.0*	77.6*
CAT	.21	.03	.06	.28	4.22*	7.86*	9.50*	16.2*
CUV	.35	.46	1.11	1.92	14.1*	30.7*	39.1*	76.9*
KO	.08	.95	2.58	.44	2.56	6.68*	10.3*	15.0*
DIS	.78	.03	.08	.14	6.40*	16.1*	20.7*	29.9*
DD	.53	.01	.02	.21	3.71	8.51*	12.8*	25.5*
EK	.31	.25	1.27	1.07	.33	2.40	3.50	5.27*
XON	1.90	.47	.34	.29	1.34	3.96*	6.32*	11.9*
GE	.30	.06	.01	.32	7.23*	20.5*	28.2*	49.2*
GM	1.02	.06	.01	.06	5.54*	12.8*	16.2*	22.7*
GT	1.18	3.84*	3.34	2.01	2.50	4.56*	7.11*	12.4*
IBM	.01	.71	.71	.10	2.82	7.80*	12.6*	17.7*
IP	.02	.18	.74	3.61	3.07	7.83*	11.4*	21.7*
MCD	1.82	.40	.48	.58	17.9*	46.6*	71.5*	105.4*
MRK	1.98	.10	1.54	.28	2.13	9.28*	16.5*	33.2*
MMM	.02	.19	.04	.90	2.44	8.89*	13.6*	25.8*
JPM	.00	.42	.34	.02	1.50	4.16*	5.39*	7.55*
MO	.05	.75	.07	.02	2.70	6.48*	10.7*	19.8*
PG	.37	.14	.04	.80	1.84	1.87	2.24	4.62*
S	.08	.18	.25	1.66	.40	2.25	5.28*	13.5*
TX	.07	2.86	2.93	2.30	9.53*	13.4*	20.4*	36.8*
UK	1.56	1.71	.90	.42	11.2*	13.8*	21.8*	28.4*
UTX	.01	1.04	.78	.31	6.54*	13.6*	19.4*	31.2*
WX	.53	1.08	1.57	1.03	1.73	4.05*	3.84*	7.02*
Z	.03	.58	.75	.18	.24	1.05	1.62	8.61*

NOTE: * indicates significant at 5% level.

Table 5. LM Test for Stock Returns for July 1962–December 1994

	m				m			
	30	50	80	100	30	50	80	100
Stock	Returns				Squared returns			
ATT	.05	.05	.1	.70	18.6*	30.7*	53.5*	67.0*
ALD	.17	.01	1.0	.15	.45	1.98	4.09*	6.60*
AA	2.98	2.16	1.3	2.83	6.66*	18.1*	36.0*	53.1*
AXP	.58	2.24	.0	.31	1.59	5.12*	8.64*	9.61*
BS	1.74	1.15	.5	.28	37.8*	62.1*	97.2*	130.2*
BA	.33	1.81	1.2	1.28	31.1*	67.0*	114.7*	167.2*
CAT	.14	.12	.0	.18	1.55	7.51*	13.2*	18.6*
CHV	.58	.08	.1	.65	29.4*	50.7*	101.4*	148.6*
KO	.81	.25	1.3	.06	3.22	9.56*	19.9*	28.9*
DIS	.04	.93	.4	.04	6.09*	12.3*	31.0*	43.0*
DD	.23	.00	.4	.58	14.5*	30.8*	51.2*	74.5*
EK	.00	.40	1.5	1.32	1.50	4.25*	9.68*	14.7*
XON	.19	3.61	2.2	2.20	2.10	5.10*	11.8*	18.8*
GE	.43	.03	.6	.46	6.70*	18.8*	42.2*	61.5*
GM	.43	1.24	.1	.51	13.8*	26.0*	46.3*	58.7*
GT	.32	1.36	2.0	1.14	4.32*	9.80*	15.4*	20.1*
IBM	.14	.10	1.1	.24	2.01	6.76*	18.0*	25.0*
IP	.21	.00	1.3	4.14*	1.89	7.68*	16.4*	27.0*
MCD	.19	.05	.1	.65	18.3*	47.1*	120.9*	167.1*
MRK	1.10	1.45	.4	.02	.78	4.42*	17.2*	35.4*
MMM	.30	.22	1.1	.96	.55	4.52*	14.1*	25.2*
JPM	.22	.23	.5	.00	1.85	3.63	7.79*	10.4*
MO	.09	.37	1.0	.26	2.36	7.10*	17.2*	25.8*
PG	.65	.18	.0	1.08	2.65	4.67*	4.51*	7.46*
S	.15	.05	.0	.23	4.88*	12.1*	21.6*	34.8*
TX	.00	1.14	2.2	2.34	16.1*	31.8*	45.5*	71.0*
UK	1.89	3.33	2.1	1.77	21.5*	32.2*	47.7*	73.0*
UTX	.06	.03	.3	.45	16.6*	40.7*	66.9*	88.2*
WX	.04	.35	.2	.90	4.85*	7.52*	11.1*	15.9*
Z	.21	.06	.0	.02	.36	5.04*	7.88*	13.6*

NOTE: * indicates significant at 5% level. Notice that AXP starts 770518, MCD starts 660705, and JPM 690401.

The third cause is a seasonal long-memory component in the returns. This case was analyzed by Lobato (1997). For an exchange-rate series (British pound against Deutsche mark for 1989 to 1994), it is shown how the presence of a strong cyclic component of about two weeks in their returns can produce spurious evidence of long memory in the squared returns. For the S&P 500, however, this explanation does not seem applicable.

The fourth cause involves size distortions. Lobato and Robinson (1997) showed that the LM test suffers from severe size distortions in the AR(1) case when the autoregressive coefficient takes values near 1; the closer to 1, the greater the distortion. In Table 6 we report a Monte Carlo study to demonstrate this possibility. We generate 5,000 replications of a first-order autoregressive conditional heteroscedasticity [ARCH(1)] process

$$y_t = \varepsilon_t \sigma_t, \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 \tag{2.1}$$

with sample size $= 1{,}000$, $\alpha_0 = .1$, five values for α_1 (.0, .3, .6, .9, .95), and where ε_t is independently and identically distributed N(0, 1). We consider three values for m, 40, 70, and 100, and report the percentage of rejections based on the 5% and 1% critical values of χ_1^2. There is some spurious evidence of long memory for $\alpha_1 = .9$ and .95 when $m = 100$. When m is smaller, this phenomenon is not so marked.

Notice that, if $|\alpha_1| > 1/\sqrt{3}$, the fourth moment is not finite. In this case our test procedure does not work.

The fifth cause is the nonexistence of higher-order moments. This is motivated by the preceding comment on the existence of the fourth moment. The LM test, as well as the Wald test, for long memory assumes that the examined series has a finite fourth moment. Loretan and Phillips (1993) argued that the fourth moment may not exist for financial series. We do not know, however, of a robust test for the nonexistence of moments in a long-memory environment. A robust procedure analyzed by Hsing (1991) is valid for weak dependent processes but not for long-memory processes. At any rate, there is no consensus in this matter. For different series, different results have been found; see Brock and de Lima (1996).

3. DISCUSSION

In this article we have examined the presence of long memory in daily stock returns and their squares using a semiparametric procedure that is robust to the presence of weak dependence. Our test results indicate no evidence of long memory in the levels of the returns. For the squared returns, however, the test results favor long memory and hence confirm the conclusion of Ding et al. (1993). Furthermore, our analysis suggests that this evidence in favor of long memory is real, not spurious.

Table 6. LM Percentage of Rejections

	m					
	40		70		100	
α	5%	1%	5%	1%	5%	1%
			A. Returns			
0	.025	.006	.034	.007	.036	.009
.3	.024	.006	.034	.007	.037	.009
.6	.023	.007	.034	.007	.046	.011
.9	.023	.006	.059	.019	.110	.046
.95	.024	.007	.066	.024	.127	.060
			B. Squared returns			
0	.026	.008	.033	.007	.036	.008
.3	.026	.009	.038	.011	.063	.022
.6	.028	.010	.054	.025	.169	.084
.9	.028	.011	.107	.052	.331	.208
.95	.028	.013	.118	.063	.353	.225

NOTE: Series follow ARCH(1) as stated in Equation (2.1). Sample size = 1,000. Number of replications = 5,000

If there is indeed long memory in the squares of the returns, then the standard statistical tools for inference are not valid (see, for instance, Beran 1994, chap. 1). In particular, inferences about squared returns and volatility using standard techniques can be misleading. For instance, the standard errors for the estimates of the coefficients of conventional ARCH or stochastic volatility models will be incorrect and hence the confidence intervals for predictions.

In the case of long memory in squared stock returns, dependence in stock returns is not properly measured by autocorrelations. In the case of the S&P 500, however, the Box and Ljung modified Q test statistic does detect dependence when using a small number of lags; this is due to the first autocorrelation. In other cases, more refined tests of independence than those based on the spectrum may be needed (see, for instance, Robinson 1991; Skaug and Tjostheim 1993; Delgado 1996; Pinkse in press, and references therein).

We also investigated intradaily stock returns for long memory because these data are now commonly used in finance (Stoll and Whaley 1990). In particular, we tested the minute-by-minute returns and their squares for IBM, one of the most heavily traded stocks. The results showed no evidence of long memory in the returns but strong evidence for the squared returns. In this article, however, we do not present our findings. The reason is that nonstationarity poses a serious problem in the case of intradaily returns. It is a well-known institutional fact that intradaily squared returns are nonstationary. For a period of a day, the time series of intradaily squared returns has an inverse J shape (Brock and Kleidon 1992; Madhavan, Richardson, and Roomans 1994). Splitting the intradaily data into what appear to be stationary periods does not permit us to test for long memory because the stationary periods are too short for the distribution of the test statistic we employ to be well approximated by its asymptotic normal distribution. One approach to treating the small-sample problem is to patch days together omitting, say, the first 10 minutes of each day; see, for example, Stoll and Whaley (1990). It is highly questionable whether this is a satisfactory solution to the nonstationarity problem.

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APPENDIX: AUTOCOVARIANCE OF SQUARES

For simplicity, let $z_t = y_t^2$ and $\bar{z}$ be its mean. Now, because $z_t - \sigma_t^2$ is a uniformly integrable zero mean independent sequence, we can apply a weak law of large numbers (WLLN) to that sequence so that

$$\bar{z} - E\bar{z} = o_p(1). \tag{A.1}$$

Now, using $(z_t - \bar{z}) = (z_t - \sigma_t^2) - (\bar{z} - \sigma_t^2)$, we get

$$\begin{aligned}\hat{\gamma}_{z,j} = \frac{1}{N}\sum_{t=1}^{N-j} [&(z_t - \sigma_t^2)(z_{t+j} - \sigma_{t+j}^2) \\ &- (z_t - \sigma_t^2)(\bar{z} - \sigma_{t+j}^2) \\ &- (z_{t+j} - \sigma_{t+j}^2)(\bar{z} - \sigma_t^2) \\ &+ (\bar{z} - \sigma_t^2)(\bar{z} - \sigma_{t+j}^2)].\end{aligned} \tag{A.2}$$

Now, as

$$\begin{aligned}\bar{z} - \sigma_t^2 &= \bar{z} - E\bar{z} + \frac{1}{N}\sum_{k=1}^{N}(\sigma_k^2 - \sigma_t^2) \\ &= \frac{1}{N}\sum_{k=1}^{N}(\sigma_k^2 - \sigma_t^2) + o_p(1),\end{aligned} \tag{A.3}$$

the last term in (A.2) is

$$\frac{1}{N^3}\sum_{t=1}^{N-j}\sum_{k=1}^{N}\sum_{k'=1}^{N}(\sigma_k^2\sigma_{k'}^2 - \sigma_k^2\sigma_{t+j}^2 - \sigma_t^2\sigma_{k'}^2 + \sigma_t^2\sigma_{t+j}^2) + o_p(1).$$

Furthermore,

$$\sum_{k=1}^{N}(\sigma_k^2 - \sigma_t^2) = K_1 I(1 \le t \le N_b) + K_2 I(N_b + 1 \le t \le N),$$

where K_1 and K_2 are given by $K_1 = (N - N_b)(\sigma_2^2 - \sigma_1^2)$ and $K_2 = -N_b(\sigma_2^2 - \sigma_1^2)$ and $I(A)$ is the indicator function; that is, $I(A) = 1$ if A is true, $I(A) = 0$ otherwise. Then the last term of (A.2) is

$$\begin{aligned}\frac{1}{N^3}\sum_{t=1}^{N-j} [&K_1^2 I(1 \le t \le N_b; 1 \le t+j \le N_b) \\ &+ K_1K_2 I(1 \le t \le N_b; N_b + 1 \le t+j \le N) \\ &+ K_1K_2 I(N_b + 1 \le t \le N; 1 \le t+j \le N_b) \\ &+ K_2^2 I(N_b + 1 \le t \le N; N_b + 1 \le t+j \le N)] \\ &+ o_p(1).\end{aligned}$$

The first summand is $K_1^2(N_b - j)I(j \le N_b)$, the third is 0, the fourth is $K_2^2(N - N_b - j)I(j \le N - N_b)$, and the second is

$$\begin{cases} jK_1K_2 & \text{if } 1 \le j \le N_b \\ N_bK_1K_2 & \text{if } N_b \le j \le N - N_b \\ (N-j)K_1K_2 & \text{if } N - N_b \le j \end{cases}$$

so that the last term in (A.2) has a different value depending on j. For $1 \le j \le N_b$, it is

$$\Psi\theta(1-\theta) - \frac{j}{N}\,\Psi(1-\theta+\theta^2),$$

for $N_b < j \le N - N_b$ it is

$$-\frac{j}{N}\,\Psi\theta^2,$$

and for $N - N_b < j \le N - 1$ it is

$$-\Psi\theta(1-\theta)\left(1 - \frac{j}{N}\right),$$

with $\Psi = (\sigma_2^2 - \sigma_1^2)$ and $\theta = N_b/N$. Now the first term in (A.2) is $o_p(1)$, applying a WLLN to the uniformly integrable zero mean independent sequence $(z_t - \sigma_t^2)(z_{t+j} - \sigma_{t+j}^2)$, and the second and third terms are $o_p(1)$ using (A.1) and (A.3).

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Comment

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This article throws further light on one of the more interesting puzzles concerning speculative markets—why do measures of volatility appear to have the long-memory property? I find it easiest to think about this result in the representation $r_t = (\text{sign } r_t)|r_t|$, where r_t takes the value of 1 if $r_t > 0$ and -1 if $r_t < 0$. Suppose that sign r_t and $|r_t|$ are independent and that sign r_t is short-memory, as shown by Granger and Ding (1995); then r_t will be short-memory even though $|r_t|$ and thus r_t^2 is long-memory, as found here and in several other works. An aspect of the article that I

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